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## On domain-wall/QFT dualities in various dimensions

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**Abstract.** We investigate domain-wall/quantum field-theory correspondences in various dimensions. Our general analysis covers not only the well studied cases in 10 and 11 dimensions, but also enables us to discuss new cases like a type I/heterotic 6-brane in 10 dimensions and domain-wall dualities in lower than 10 dimensions. The examples we discuss include ‘d-branes’ in six dimensions preserving eight supersymmetries and extreme black holes in various dimensions. In the latter case we construct the quantum mechanics Hamiltonian and discuss several limits.

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### Introduction

Anti-de Sitter (AdS) gravity has attracted much attention due to the conjectured correspondence to a conformal field theory (CFT) on the boundary of the AdS spacetime [1] (for a review, see [2]). This CFT is given by the worldvolume field theory, in the limit that gravity is decoupled, of a brane moving in the AdS background. In order for this correspondence to hold, it is essential to have an AdS space, whose isometry group acts as the conformal group on the boundary [3, 4]. The regular  $p$ -branes (i.e. the D3-brane in  $D = 10$  dimensions and the M2/M5-branes in  $D = 11$  dimensions) do indeed exhibit an AdS spacetime in the so-called near-horizon limit. However, for the other branes both the string as well as the Einstein metric do not lead to a similar anti-de Sitter geometry.

On the other hand, for all supersymmetric  $p$ -branes there does exist a special frame in which the  $p$ -brane geometry, in the near-horizon limit, factorizes into a sphere times an AdS or flat spacetime [5, 6]. Moreover, the near-horizon geometry of these branes contains a non-trivial dilaton. Choosing a particular radial coordinate, the dilaton field is in fact linear in that radial coordinate of AdS spacetime. The metric of this special frame is conformally equivalent to the string frame metric and coincides with the so-called sigma-model metric coupling to a *dual* brane probe. It is therefore called the ‘*dual* frame’ metric [5]. AdS spaces can be viewed as special cases (in the sense of having a zero dilaton) of AdS spaces with a linear dilaton. AdS spaces with a linear dilaton are, in fact, conformally equivalent to domain-wall (DW) spacetimes [7]. The presence of the dilaton breaks the scale invariance and the AdS isometry group gets reduced to the Poincaré isometry group of the DW.

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Since all branes are related by duality, it is natural to conjecture that the AdS/CFT dualities based on the regular  $p$ -branes can be generalized to so-called domain-wall dualities, corresponding to all the other  $p$ -branes, where the CFT is replaced by an ordinary QFT. Indeed, for the  $Dp$ -branes ( $p = 0, \dots, 6$ ) in  $D = 10$  dimensions such DW/QFT dualities have been discussed in [8, 9]. Due to the holographic property of an AdS space or of a flat spacetime with linear dilaton [10], the dual frame is the natural basis for a discussion of this DW/QFT correspondence [9]. We note that the QFT is not conformally or scale invariant. There will be a non-trivial renormalization group flow and only at fixed points (in the UV and/or IR) can we expect a restoration of the conformal symmetry. Examples of such fixed points are the limits where  $Dp$ -branes decompactify to M-branes [8, 11].

The purpose of this paper is to extend the discussion of the DW/QFT correspondence, given in [8, 9] for  $Dp$ -branes in 10 dimensions, to general two-block  $p$ -branes in various dimensions. All the  $p$ -branes we consider have a near-horizon geometry of the form  $DW \times S$  (domain wall times sphere). The  $p$ -branes in  $D < 10$  dimensions can be viewed as  $p$ -branes in  $D = 10$  (or intersections of such branes with the harmonics identified) reduced over some compact manifold  $K$ . From the  $D = 10$  point of view these  $p$ -branes have a near-horizon geometry of the form  $DW \times S \times K$ . More general near-horizon geometries of the form  $DW \times S \times S \times \mathbb{E}$  (domain wall times sphere times sphere times Euclidean space) can be obtained by considering intersections where the harmonics of the participating branes have not been identified. Such intersections and their near-horizon geometries have been studied in the second reference of [6] and in [12].

We will start our analysis with a general discussion applicable to any  $p$ -brane with a non-zero dilaton. We discuss the so-called field-theory limit, i.e. the low-energy limit in which gravity decouples from the worldvolume QFT. This limit is defined such that the QFT contains at least one fixed coupling constant. Our analysis shows that, in order to obtain a well behaved DW/QFT correspondence, we need to restrict ourselves to a subclass of  $p$ -branes. Our choice of restriction is motivated by the condition that the so-called holographic energy (or supergravity probe) scale  $u$  is related to the string energy (or D-brane probe) scale  $U$  via fixed quantities only. The exact relation can be found in equation (63). This condition leads to two constraints, whose precise form can be found in (64). These constraints tell us to which  $p$ -branes we should restrict ourselves and which coupling constant in the worldvolume field theory, i.e. corresponding to scalars or vectors, etc, should be kept fixed. We find that these constraints are sufficient to obtain well behaved DW/QFT dualities. The restricted class of  $p$ -branes is distinguished by the fact that their effective tension is proportional to the inverse string coupling constant. In fact, those in  $D < 10$  can be interpreted as intersections of D-branes in  $D = 10$  which are reduced over relative transverse directions only. The branes participating in these intersections are equal in number and delocalized in the compact relative transverse directions.

We will discuss in more detail various special cases in ten as well as in lower dimensions. These special cases include a type I/heterotic 6-brane in 10 dimensions, the D8-brane, the so-called ‘d-branes’ in six dimensions and the extreme black holes in various dimensions. In this paper we will not discuss the details of the worldvolume theory, all that we need is the worldvolume field content. An exception is made for the extreme black holes where we will present the (generalized conformal) quantum mechanics Hamiltonian and discuss several limits.

The paper is organized as follows. In section 1 we summarize the relationship between DW and AdS spacetimes. In section 2 we calculate the near-horizon geometries of a generic  $p$ -brane. In section 2.1 we show that for all  $p$  the near-horizon geometries factorize into a DW times sphere geometry using the dual-frame metric. In the next subsection we consider the

reduction over the spherical part of the near-horizon geometry and relate the DW times sphere near-horizon geometry to a DW spacetime after reduction. In a third subsection we discuss the special cases in which the DW part of the near-horizon geometry becomes conformally flat or pure AdS (with zero dilaton). In section 3 we discuss the field-theory limit for the general case. The formulae we give in this section are applied to the special cases we discuss in the remaining sections. First, in section 4 we discuss the ten-dimensional  $Dp$ -branes, including the D8-brane, and the so-called six-dimensional ‘ $d$  $p$ -branes’ preserving eight supersymmetries. Next, in section 5 we discuss the quantum mechanics of 0-branes, or extreme black holes, in various dimensions, and in section 6 we investigate the special case of a type I/heterotic 6-brane in 10 dimensions. Section 6 is a bit special in the sense that it can be read independently of the rest of the paper. In section 7 we give our conclusions. We have included three appendices. In appendix A we give our  $p$ -brane charge conventions; in appendix B we elaborate on the dual-frame metric and show that it is identical to the sigma-model metric provided that one uses the sigma model of the dual brane. Finally, appendix C contains a glossary of most of the symbols used in this paper together with a short description and the equation where the symbol is first used.

## 1. Domain walls and anti-de Sitter spacetimes

Before we discuss in the next section the near-horizon geometries of a general  $p$ -brane, we will first discuss in this section the geometries of domain wall<sup>†</sup> and anti-de Sitter spacetimes. It is known that AdS spaces are special cases of domain-wall spaces characterized by the absence of a non-trivial dilaton background [7]. We will show this explicitly in this section.

Domain-wall spaces solve the equations of motion obtained by varying a (super)gravity action with a cosmological constant  $\Lambda$  and a dilaton<sup>‡</sup>. They correspond to  $p$ -branes with worldvolume dimension  $d = p + 1$  which is one less than the dimension  $D$  of the target spacetime they live in. The part of the supergravity action needed to describe the DW solution is given by (we use the Einstein frame)

$$S_{d+1}^E = \int d^{d+1}x \sqrt{g} \left[ R - \frac{4}{d-1} (\partial\phi)^2 + e^{-b\phi} \Lambda \right], \quad (1)$$

where  $b$  is an arbitrary dilaton coupling parameter.

Performing a Poincaré dualization, which replaces the cosmological constant  $\Lambda$  by a  $(d+1)$ -form field strength  $F_{d+1}$ , allows us to discuss naturally objects of codimension one coupling to a  $d$ -form potential, defining a domain wall. In terms of the field strength  $F_{d+1}$  the action is given by

$$S_{d+1}^E = \int d^{d+1}x \sqrt{g} \left[ R - \frac{4}{d-1} (\partial\phi)^2 - \frac{1}{2(d+1)!} e^{b\phi} F_{d+1}^2 \right]. \quad (2)$$

The equations of motion following from the action (2) can be solved using the general

<sup>†</sup> For earlier work on domain-wall solutions in  $d = 4$  supergravity theories, see [13].

<sup>‡</sup> We use conventions in which our metric is mostly plus. The conventions used imply that when  $\Lambda > 0$  we have an AdS vacuum and when  $\Lambda < 0$  we have a de Sitter vacuum (neglecting the dilaton).

$p$ -brane ansatz involving harmonic functions. The solutions are (in the Einstein frame)<sup>†</sup>

$$\begin{aligned} ds_E^2 &= H^{-4\varepsilon/(d-1)\Delta_{DW}} dx_d^2 + H^{-4\varepsilon d/(d-1)\Delta_{DW}-2(\varepsilon+1)} dy^2, \\ e^\phi &= H^{-(d-1)b\varepsilon/4\Delta_{DW}}, \\ F_{01\dots d-1 y} &= \epsilon_{01\dots d-1} \partial_y H^\varepsilon, \end{aligned} \quad (3)$$

with  $\varepsilon$  an arbitrary parameter and the parameter  $\Delta_{DW}$  defined by

$$\Delta_{DW} = \frac{(d-1)b^2}{8} - \frac{2d}{d-1}. \quad (4)$$

This  $\Delta_{DW}$  is a useful parameter since it is invariant under reductions or oxidations (in the Einstein frame) [14]<sup>‡</sup>.

The function  $H$  is harmonic on the one-dimensional transverse space with coordinate  $y$ :

$$H = c + Q|y|, \quad (5)$$

with  $c$  and  $Q$  constant. Here it is understood that the domain wall is positioned at the discontinuity  $y = 0$ , and for  $y < 0$  we are allowed to use a different value of  $Q$ . The value of  $Q$  can be expressed in terms of a mass parameter  $m$  in the following way:

$$Q\varepsilon = m, \quad (6)$$

where  $m$  is related to the cosmological constant through the equation

$$\Lambda = \frac{-2m^2}{\Delta_{DW}}. \quad (7)$$

We see that using the ansatz (3) allows for an undetermined parameter  $\varepsilon$  in the domain-wall solution. Note that the charge  $Q$  cannot be considered a physical parameter because of its dependence on  $\varepsilon$ . The origin of this parameter is in the fact that there are coordinate transformations, labelled by  $\varepsilon$ , that keep the solution within the same ansatz. The explicit form of these coordinate transformations is given in [15].

Different choices of  $\varepsilon$  lead to different expressions for the metric. For instance, one can always choose  $\varepsilon$  such that the metric is conformally flat:

$$\varepsilon = \frac{-\Delta_{DW}}{\Delta_{DW} + 2}. \quad (8)$$

This choice was made in [16] for the domain wall in  $d+1 = 10$  dimensions. Another possibility is to choose a ‘D-brane basis’ in which the two powers of the harmonic functions occurring in the string frame metric are opposite:

$$\varepsilon = \frac{-\Delta_{DW}(d-1)}{b(d-1) + \Delta_{DW}(d-1) + 2(d+1)}. \quad (9)$$

This choice was made in [17] for  $d+1 = 10$  dimensions.

To adapt the discussion to what we will encounter in the next section when we discuss the near-horizon geometries, we shift the position of the domain wall to infinity, allowing us to

<sup>†</sup> In this section we take  $g_s = 1$ . Later, when we discuss the field-theory limit in section 3, we will put  $g_s$  back into the dilaton background expression.

<sup>‡</sup> Our definition of  $\Delta_{DW}$  differs from the definition in, for example, [14] (except when  $D = 10$ ). The reason for this is the fact that we use different normalizations (except when  $D = 10$ ) for the dilaton. For instance, in [14] in any dimension there is a  $\frac{1}{2}$  in front of the dilaton kinetic term (using the Einstein frame). This differs from our normalization of the dilaton kinetic term, as can be seen from equation (1).

discard the constant  $c$  in the harmonic function. Furthermore, we get rid of the free parameter  $\varepsilon$  by making the following  $y \rightarrow \lambda$  coordinate transformation:

$$Qy = e^{-Q\lambda}. \quad (10)$$

The domain-wall solution in the new  $\lambda$  coordinate reads

$$\begin{aligned} ds_E^2 &= \exp \left[ m\lambda \left( \frac{(d-1)b^2}{4\Delta_{DW}} \right) \right] \left( \exp \left[ -2m\lambda \left( \frac{2+\Delta_{DW}}{\Delta_{DW}} \right) \right] dx_d^2 + d\lambda^2 \right), \\ e^\phi &= e^{(d-1)bm\lambda/4\Delta_{DW}}. \end{aligned} \quad (11)$$

This solution corresponds to the action (1) with  $\Lambda$  given by (7).

Written like this, it is clear that we can perform a conformal transformation to get rid of the conformal factor in front of the metric. We thus obtain the regular ‘dual-frame’ metric  $g_D$

$$g_D = e^{-b\phi} g_E, \quad (12)$$

with

$$ds_D^2 = e^{-2m\lambda[(2+\Delta_{DW})/\Delta_{DW}]} dx_d^2 + d\lambda^2, \quad e^\phi = e^{(d-1)bm\lambda/4\Delta_{DW}}. \quad (13)$$

Note that when  $\Delta_{DW} = -2$  the metric becomes conformally flat.

In the dual frame (12) the domain-wall metric (13) describes an  $\text{AdS}_{d+1}$  space. In fact, this metric has constant negative curvature which, however, is not fixed by the cosmological constant. The deviation is parametrized by the free dilaton coupling parameter  $b$  corresponding to a non-vanishing dilaton charge. Furthermore, as discussed in [5], the linear dilaton corresponds to a non-vanishing conformal Killing vector, which means that conformal transformations have to be accompanied by a shift in the dilaton, i.e. yield a running coupling constant.

## 2. Near-horizon geometries of $p$ -branes

In this section we discuss the near-horizon geometry of a generic  $p$ -brane. In section 2.1 we calculate the near-horizon geometry in terms of the dual-frame metric. In subsection 2.2 we discuss the reduction over the spherical part of the near-horizon metric. Finally, in section 2.3 we discuss as special cases the conformally flat and  $\text{AdS}$  near-horizon geometries.

### 2.1. $p$ -branes in the dual frame

Our starting point is the  $D$ -dimensional action

$$S_D = \int d^D x \sqrt{g} \left[ R - \frac{4}{D-2} (\partial\phi)^2 - \frac{1}{2(\tilde{d}+1)!} e^{-a\phi} F_{\tilde{d}+1}^2 \right], \quad (14)$$

which contains three independent parameters: the target spacetime dimension  $D$ , the dilaton coupling parameter  $a$ , and a parameter  $p$  specifying the rank  $D-p-2$  of the field strength  $F$ . Furthermore, we have introduced two useful dependent parameters  $d$  and  $\tilde{d}$  which are defined by

$$\begin{aligned} d &= p+1 && \text{dimension of the worldvolume,} \\ \tilde{d} &= D-d-2 && \text{dimension of the dual-brane worldvolume.} \end{aligned} \quad (15)$$

We next consider the following class of diagonal ‘two-block’  $p$ -brane solutions (using the Einstein frame)<sup>†</sup>:

$$\begin{aligned} ds_E^2 &= H^{-4\tilde{d}/(D-2)\Delta} dx_d^2 + H^{4d/(D-2)\Delta} dx_{\tilde{d}+2}^2, \\ e^\phi &= H^{(D-2)a/4\Delta}, \\ F &= *(dH^{-1} \wedge dx_1 \wedge \cdots \wedge dx_d), \end{aligned} \quad (16)$$

where  $*$  is the Hodge operator on  $D$ -dimensional spacetime and  $\Delta$  a generalization of the  $\Delta_{DW}$  in the previous section defined by<sup>‡</sup>

$$\Delta = \frac{(D-2)a^2}{8} + \frac{2d\tilde{d}}{D-2}, \quad (17)$$

which is, in contrast to the dilaton coupling parameter  $a$ , invariant under reductions and oxidations (in the Einstein frame). The function  $H$  is harmonic over the  $\tilde{d} + 2$  transverse coordinates and, assuming that

$$\tilde{d} \neq -2, 0, \quad (18)$$

(i.e. no constant or logarithmic harmonic) this harmonic function is given by

$$H = 1 + \left(\frac{r_0}{r}\right)^{\tilde{d}}, \quad (19)$$

where  $r_0$  is an arbitrary integration constant with the dimensions of length, which is of course related to the charge (and mass) of the  $p$ -brane (see appendix A).

The two-block solutions (16) include the (supersymmetric) domain-wall spaces of the previous section. They correspond to the case where  $\tilde{d} = -1$ ,  $\epsilon = -1$  and  $r_0 = 1/m$ . The solutions also include the known branes in 10 and 11 dimensions as well as branes in lower dimensions. In table 1 we give the values of  $a$  and  $\Delta$  of a few cases, including the well-known cases of the M2-brane, the M5-brane, the D $p$ -branes ( $p = 0, 1, \dots, 6, 8, 9$ ), the fundamental string, or NS1-brane, the solitonic 5-brane, or NS5-brane, as well as the ‘d-branes’ in six dimensions and the (supersymmetric) dilatonic black holes in four dimensions. For these cases the parameters  $a$  and  $\Delta$  are given by table 1.

**Table 1.** This table indicates the values of  $a$  and  $\Delta$  (in the Einstein frame) for a variety of branes in diverse dimensions.

Dimension	$a$	$\Delta$	Name
11	0	4	M-branes
10	$(3-p)/2$	4	D $p$ -branes
	-1	4	NS1-brane
	1	4	NS5-brane
6	$1-p$	2	d $p$ -branes
5	0	4/3	m-brane
4	$2\sqrt{3}$	4	Black hole
	2	2	”
	$2/\sqrt{3}$	4/3	”
	0	1	RN black hole

<sup>†</sup> We work with *magnetic* potentials.

<sup>‡</sup> We always have  $\Delta > 0$  except for domain walls ( $\tilde{d} = -1$ ) when  $\Delta$  can be negative. Note that  $\Delta$  is invariant under toroidal reduction but not under sphere reduction. In subsection 2.2 we will perform such sphere reductions and obtain domain walls with  $\Delta_{DW} < 0$  out of branes with  $\Delta > 0$ .

If the branes under consideration preserve any supersymmetries we can set [14]

$$\Delta = \frac{4}{n}, \quad (20)$$

where generically  $32/2^n$  is the number of unbroken supersymmetries. In a case of intersecting branes embedded in a theory with 32 supersymmetries  $n$  is the number of branes that participate in the intersection<sup>†</sup>

The two-block solutions (16) do *not* involve waves and Kaluza–Klein monopoles. We also excluded D-instantons, whose near-horizon geometry has been discussed in [18, 19]. The case where  $\tilde{d} = 0$  that is excluded by (18) corresponds to branes with two transverse directions, e.g. the D7-brane in  $D = 10$ . The other case  $\tilde{d} = -2$ , excluded by (18), corresponds to spacetime-filling branes. Such branes are given by a flat Minkowski spacetime solution and have no near-horizon geometries.

We next consider the limit for which the constant part in the harmonic function is negligible, i.e. for the limits

$$\begin{aligned} \frac{r}{r_0} &\rightarrow \infty & \text{for } \tilde{d} = -1, \\ \frac{r}{r_0} &\rightarrow 0 & \text{all other cases.} \end{aligned} \quad (21)$$

Assuming that the branes are positioned at  $r = 0$  this limit brings us close to the brane when  $\tilde{d} > 0$ . When  $\tilde{d} = -1$ , however, this limit takes us far away from the brane. We refer to this limit as the near-horizon limit.

In this limit the metric and dilaton becomes

$$ds_E^2 = \left(\frac{r_0}{r}\right)^{-4\tilde{d}^2/\Delta(D-2)} dx_d^2 + \left(\frac{r_0}{r}\right)^{4d\tilde{d}/\Delta(D-2)} dx_{\tilde{d}+2}^2, \quad e^\phi = \left(\frac{r_0}{r}\right)^{(D-2)a\tilde{d}/4\Delta}. \quad (22)$$

As in the previous section we go to the dual frame, which is defined by the conformal rescaling

$$g_D = e^{(a/\tilde{d})\phi} g_E. \quad (23)$$

In this dual frame the spherical part factors off in the near-horizon limit as in [9].

After the transition of the Einstein metric to the regular ‘dual-frame’ metric in the action (14) all terms in the action are multiplied by the *same* dilaton factor:

$$S_D = \int d^D x \sqrt{g} e^{\delta\phi} \left[ R + \gamma (\partial\phi)^2 - \frac{1}{2(\tilde{d}+1)!} F_{\tilde{d}+1}^2 \right] \quad (24)$$

with

$$\delta = -\frac{(D-2)a}{2\tilde{d}}, \quad \gamma = \frac{D-1}{D-2} \delta^2 - \frac{4}{D-2}. \quad (25)$$

The metric in the dual frame is given by

$$ds_D^2 = \left(\frac{r_0}{r}\right)^{2(1-2\tilde{d}/\Delta)} dx_d^2 + \left(\frac{r_0}{r}\right)^2 dr^2 + r_0^2 d\Omega_{\tilde{d}+1}^2. \quad (26)$$

Redefining the radius by

$$\left(\frac{r_0}{r}\right) = e^{-\lambda/r_0}, \quad (27)$$

<sup>†</sup> Sometimes branes can be added without breaking more supersymmetries, in which case the above rule about the number of unbroken supersymmetries and the number of intersecting branes involved does not hold.



we can write the near-horizon metric ( $\lambda \rightarrow +\infty$  for domain walls and  $\lambda \rightarrow -\infty$  for all other branes) as

$$ds_D^2 = e^{-2(1-2\tilde{d}/\Delta)\lambda/r_0} dx_d^2 + d\lambda^2 + r_0^2 d\Omega_{\tilde{d}+1}^2, \quad \phi = -\frac{(D-2)a\tilde{d}}{4\Delta} \frac{\lambda}{r_0}. \quad (28)$$

We see that the spacetime factorizes into a spherical part and a linear dilaton part.

To make contact with other results (e.g. [1, 9]) it is useful to write the dual-frame solution (26) as

$$ds_D^2 = r_0^2 \left[ \left( \frac{u}{\mathcal{R}} \right)^2 dx_d^2 + \left( \frac{\mathcal{R}}{u} \right)^2 du^2 + d\Omega_{\tilde{d}+1}^2 \right], \quad (29)$$

$$e^\phi = r_0^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]} \left( \frac{u}{\mathcal{R}} \right)^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]},$$

where we made the  $r \rightarrow u$  coordinate transformation:

$$\frac{u}{\mathcal{R}} = \frac{r^\beta}{r_0^{\beta+1}}, \quad \mathcal{R} = \frac{1}{\beta} \quad (30)$$

with  $\beta$  given by

$$\beta = \frac{2\tilde{d}}{\Delta} - 1. \quad (31)$$

The horospherical coordinate  $u$  can be interpreted as the energy scale describing supergravity probes [11]. In the metric (29) the point  $u = 0$  is a non-singular Killing horizon<sup>†</sup> and  $u = \infty$  corresponds to the boundary of  $AdS_{d+1}$ .

Using the horospherical coordinate  $u$  we recognize the metric as that of an AdS space times a sphere. Through (30) we see that  $\mathcal{R}$  is the size of the AdS part relative to the sphere. As expected we recover in the cases of D3, M2 and M5 the well known relations between the radius of the sphere and the radius of AdS, being, respectively,  $\mathcal{R} = 1, \frac{1}{2}$  and 2. The dimensionful length scale  $\mathcal{R}r_0$  is the radius of the hyperboloid surface embedded in a  $(p+2)$ -dimensional flat spacetime with  $SO(2, p)$  isometry, giving the (induced) metric (29).

Summarizing, in this subsection we have shown that in the dual frame, defined by (23), all  $p$ -branes have a near-horizon geometry that factorizes into a domain-wall spacetime  $DW_{d+1}$  times a sphere  $S^{\tilde{d}+1}$ . Note that the domain-wall part of the metric has all the isometries of an AdS space. These isometries are broken in the complete background because of a non-trivial linear dilaton.

## 2.2. Reduction over the sphere

Sphere reductions of supergravities that admit domain wall  $\times$  sphere vacuum solutions (as opposed to AdS  $\times$  sphere vacuum solutions) have only been recently discussed in the literature [9, 20]. We expect that when we reduce over the  $\tilde{d}+1$  angular variables of the sphere we will end up with a gauged supergravity in  $d+1$  dimensions whose action contains the terms

$$S_{d+1}^R = \int d^{d+1}x \sqrt{g} e^{\delta\phi} [R + \gamma(\partial\phi)^2 + \Lambda]. \quad (32)$$

The number of supersymmetries of this action is determined by the original  $p$ -brane. The value of  $\Lambda$  can be determined from the  $(\tilde{d}+1)$ -form curvature which, in the magnetic picture,

<sup>†</sup> Strictly speaking, the near horizon (21) does not always correspond to the limit  $u \rightarrow 0$ . An exception is the  $D = 10$  D6-brane in which case we have  $\beta < 0$  and hence the limit (21) implies the limit  $u \rightarrow \infty$ .

is just the volume form on the sphere and there is a contribution from the Ricci scalar. We will give the value of  $\Lambda$  after we have determined the domain-wall solution in the Einstein frame. Transforming to the Einstein frame<sup>†</sup>

$$ds_E^2 = e^{[2\delta/(d-1)]\phi} ds_D^2, \quad (33)$$

and rescaling the dilaton as  $\phi \rightarrow \phi/c$  with (the parameter  $\beta$  is defined in (31))

$$c^2 = \frac{(\beta + 1)\tilde{d}}{\tilde{d} - \beta}, \quad (34)$$

we obtain for the action *exactly* the expression given in (1), with the dilaton coupling parameter  $b$  given by

$$b = a \frac{(D-2)d}{(d-1)\tilde{d}} c. \quad (35)$$

The worldvolume part of the solution in this Einstein frame is given by

$$ds_E^2 = e^{b^2(d-1)(\tilde{d}-\beta)\lambda/8r_0} [e^{2\beta\lambda/r_0} dx_d^2 + d\lambda^2], \quad \phi = -\frac{(D-2)a\tilde{d}}{4\Delta} \frac{\lambda}{r_0 c}. \quad (36)$$

For the generic case this is a domain-wall solution  $DW_{d+1}$  in  $d+1$  spacetime dimensions and in order to relate it to those discussed in section 1, we must give the parameter  $\Delta_{DW}$ . We find that  $\Delta_{DW}$  is given by

$$\Delta_{DW} = \frac{-2\tilde{d}}{(\tilde{d} - \beta)}. \quad (37)$$

Using the above expression and identifying

$$m = \frac{-\tilde{d}}{r_0}, \quad (38)$$

we recover the general form for the metric given by (11). We can also use the general analysis in section 1 to determine the value of the cosmological constant  $\Lambda$  given in (7). We find

$$\Lambda = \frac{\tilde{d}}{2r_0^2} \left[ 2(\tilde{d} + 1) - \frac{4\tilde{d}}{\Delta} \right]. \quad (39)$$

The reduction of the Ricci scalar is responsible for the first term in this expression and the second term is from the reduction of the (magnetic)  $(\tilde{d}+1)$ -form curvature. Analysing this expression we find that all  $p$ -brane near-horizon geometries give a  $\Lambda > 0$  (with  $1 \leq \tilde{d} \leq (D-3)$ ) except for the domain walls, which have  $\tilde{d} = -1$  and a sign change occurs, giving  $\Lambda < 0$ . We note that this is not in contradiction with the fact that all  $p$ -branes (including the domain walls) have AdS geometries in the near-horizon limit (as defined in section 2), because the linear dilaton will also contribute to an *effective* cosmological constant which is always positive.

Notice that the right-hand side of (37) only depends on  $\tilde{d}$ . This expresses the fact that when we perform a double-dimensional reduction (under which  $\tilde{d}$  is invariant), the near-horizon geometries of the  $p$ - and  $(p-1)$ -brane are also related by a double-dimensional reduction. In contrast,  $\Delta_{DW}$  is not invariant under a direct dimensional reduction ( $\tilde{d} \rightarrow \tilde{d}-1$ ): the near-horizon domain-wall solution is mapped to another near-horizon domain-wall solution with  $\tilde{d}$  replaced by  $\tilde{d}-1$ .

For the convenience of the reader we give tables 2–4 in which the value of  $\Delta_{DW}$  belonging to different  $p$ -branes are given.

<sup>†</sup> This transformation to the Einstein frame cannot be performed for  $p = 0$ .

**Table 2.** The values of  $\Delta_{DW}$  for the different ten-dimensional Dp-branes.

Brane	D0	D1/F1	D2	D3	D4	D5/NS-5	D6	D8
$\Delta_{DW}$	$-\frac{28}{9}$	-3	$-\frac{20}{7}$	$-\frac{8}{3}$	$-\frac{12}{5}$	-2	$-\frac{4}{3}$	4

**Table 3.** The values of  $\Delta_{DW}$  for the different dp-branes in six dimensions with  $n = 2$ .

dp	d0	d1	d2	d4
$\Delta_{DW}$	-6	-4	-2	2

**Table 4.** The values of  $\Delta_{DW}$  for the different dilatonic black holes in four dimensions and the corresponding 'parent' branes in higher dimensions.

a	$2\sqrt{3}$	2	$2/\sqrt{3}$	0
$\Delta_{DW}$	-4/3	-2	-4	$\infty$
Parent	D6	d2	m1	—

### 2.3. Special cases

In this subsection we discuss two special cases where the domain wall  $DW_{d+1}$  in the near-horizon geometry becomes a (conformally) flat space  $\mathbb{R}^{d,1}$  or where the dilaton vanishes and the spacetime becomes an anti-de Sitter spacetime  $AdS_{d+1}$ .

*Conformally flat spaces.* We require that the near-horizon geometry is a flat space  $\mathbb{R}^{d,1}$  in the dual frame. From (26) we see that the condition for flat space is given by

$$2\tilde{d} = \Delta, \quad (40)$$

which, using (31), can also be expressed as  $\beta = 0$ .

This implies that

$$\Delta_{DW} = -2, \quad b^2 = \frac{16}{(d-1)^2}, \quad (41)$$

and thus the domain-wall solution  $DW_{d+1}$  becomes

$$ds_E^2 = e^{[2\tilde{d}/(d-1)]\lambda/r_0} [dx_d^2 + d\lambda^2], \quad \phi = \frac{-a\lambda}{r_0 c}, \quad (42)$$

with  $a^2 = 4\Delta^2/(D-2)^2$ .

We now look to solutions of condition (40). We restrict ourselves to supersymmetric solutions enabling us to replace  $\Delta$  by  $4/n$ . For  $n = 1$  this condition yields

$$p = D - 5, \quad (43)$$

which leads to 5-branes in 10 dimensions and all double-dimensional reductions of the 5-brane to lower dimensions. For the next case,  $n = 2$ , condition (40) yields

$$p = D - 4, \quad (44)$$

which leads to 6-branes in 10 dimensions and all double-dimensional reductions of this 6-brane in lower dimensions. Notice that  $32/2^n$  is the number of unbroken supersymmetries and therefore this must be a type I or a heterotic 6-brane. Such branes are charged under a

Yang–Mills gauge field, which is broken to an Abelian  $U(1)$  gauge field. For the heterotic case the solution in the string frame metric  $g_S$  is given by ( $a = \frac{1}{2}$ )

$$ds_H^2 = dx_d^2 + H^2 dx_{d+2}^2, \quad e^{2\phi} = H, \quad (45)$$

and the type I solution reads ( $a = -\frac{1}{2}$ )

$$ds_I^2 = H^{-1/2} dx_d^2 + H^{3/2} dx_{d+2}^2, \quad e^{-2\phi} = H. \quad (46)$$

In both cases the solution for the magnetic gauge field coincides with the solution for the type II 6-brane gauge field. Note, that the non-spherical part in both cases is not flat, but *conformally* flat near the horizon. We will discuss the type I/heterotic 6-brane in more detail in section 6.

*Anti-de Sitter spaces.* The anti-de Sitter case  $DW_{d+1} = \text{AdS}_{d+1}$  corresponds to the special case that  $a = b = 0$ . This condition yields (see equation (17))

$$\frac{2d\tilde{d}}{\Delta} = (d + \tilde{d}). \quad (47)$$

It also implies that

$$\Delta_{DW} = -2\frac{d}{d-1} \quad \text{and} \quad \beta = \frac{\tilde{d}}{d}. \quad (48)$$

The solution for these cases becomes (the overall conformal factor vanishes)

$$ds_E^2 = e^{2\beta\lambda/r_0} dx_d^2 + d\lambda^2, \quad \phi = 0. \quad (49)$$

The condition (47) can be satisfied for the cases where we preserve some amount of supersymmetry (i.e. using (20)). We find all the well-known cases as we have summarized in table 5 below [4, 6, 21].

**Table 5.** This table indicates the values of  $\beta$ ,  $\Delta_{DW}$  and  $\Delta$  (in the Einstein frame) for all branes that have an AdS near-horizon geometry.

$D$	$\beta$	$\Delta_{DW}$	$\Delta$	Name
11	$\frac{1}{2}$	$-\frac{12}{5}$	4	M5-brane
	2	-3	4	M2-brane
10	1	$-\frac{8}{3}$	4	D3-brane
6	1	-4	2	d1-brane
5	$\frac{1}{2}$	-4	$\frac{4}{3}$	m1-brane
	2	$\infty$	$\frac{4}{3}$	m0-brane
4	1	$\infty$	1	RN black hole

This table shows that there are branes with an AdS geometry for  $n = 1, \dots, 4$  and that these branes have only *three* different values for  $\beta$ :  $\frac{1}{2}$ , 1 and 2. Note that the m-branes have the same values for  $\beta$  as the M-branes. In fact, it turns out that the m1-brane is an intersection of three M5-branes and that the m0-brane is an intersection of three M2-branes. A similar thing happens for the d1 and the D3; they both have  $\beta = 1$  and the d1 is an intersection of two D3-branes.

Note that the case where  $p = 0$  is special: for  $p = 0$  the value of  $\Delta_{DW}$  blows up and using (7) we see that the cosmological constant vanishes. However, the corresponding  $\text{AdS}_2$  space still satisfies the Einstein equation in two dimensions and has a non-zero curvature.

### 3. The field-theory limit

In this section we first give a general discussion of the field-theory limit of  $p$ -branes (subsection 3.1). For the convenience of the reader we discuss in the following two subsections, as a familiar example, the pure AdS cases (subsection 3.2) and the conformally flat cases (subsection 3.3). In section 4 and 5 we will discuss examples, some of which are known but others which have not been studied before.

#### 3.1. The general case

The fact that every  $p$ -brane near-horizon metric factorizes into a domain wall times a sphere suggests that we can relate supergravities on such a background to field theories on the boundary in the (generalized) way described in [8, 9]. The conformally flat case requires a special treatment. This case will be discussed separately in subsection 3.3.

When discussing the field-theory limit, one usually starts by writing the harmonic function in terms of appropriately defined field-theory (energy and coupling) parameters, which are kept fixed, and then show that in taking the limit  $\alpha' \rightarrow 0$  (low-energy limit) the harmonic function tends to infinity, meaning that we end up in the near-horizon region. This near-horizon metric is then shown to stay finite in  $\alpha'$  units when taking the low-energy limit.

Because we want to discuss  $p$ -brane backgrounds in general, we do not want to start by defining appropriate field-theory variables (because in general this will change for every  $p$ -brane). Instead, we will proceed in the opposite direction. Starting from the near-horizon geometry, we perform the  $r \rightarrow u$  coordinate transformation given in (30). In terms of the new ‘holographic’ coordinate  $u$  the near-horizon metric is AdS for all  $p$ -branes and it is finite in  $\alpha'$  units. Afterwards we perform a consistency check and show that the low-energy limit does indeed take us into the near-horizon region.

Until now we assumed  $g_s \equiv 1$  for simplicity, but for the present discussion we must re-insert the explicit  $g_s$  dependence into the expression for the dilaton. In other words, we multiply our present expression for the dilaton background by an extra factor  $g_s$  such that  $e^{\phi(\infty)} = g_s$ . Now that we have an explicit  $g_s$  in our dilaton background we have to re-analyse the conformal transformation that takes us to the dual frame. We observe that when going to the dual frame via a conformal transformation we will be left with an overall power of  $g_s$  in the dual-frame metric (23). To avoid this we must redefine the conformal transformation with appropriate powers of  $g_s$  (see the equation below). To obtain a factor of  $\alpha'$  in front of the metric (29) instead of  $r_0^2$  we must make a further modification of the conformal transformation. Using the fact that  $\alpha' = (r_0/L)^2$  (with  $L$  defined by (A11), see the appendix), we redefine the conformal transformation to the dual frame in the following way:

$$g_D = L^{-2} \left( \frac{e^\phi}{g_s} \right)^{a/\tilde{d}} g_E. \quad (50)$$

The redefinition (50) of the conformal transformation will introduce an explicit<sup>†</sup>  $g_s$  and  $L$  dependence in the dual-frame action. These dependences are determined as follows. As the low-energy limit of string theory, the string frame solutions have no explicit dependences on  $g_s$ . In the action the gravitational + dilaton sector scales as the usual  $1/g_s^2$  (hidden in the dilaton). This should not change when we go to the Einstein frame. If we choose the metric in the Einstein frame to not carry an explicit  $g_s$  dependence (as we do in this paper), the gravitational part of the action should scale explicitly with  $1/g_s^2$  (there is no dilaton to hide it in). The same

<sup>†</sup> By explicit we mean the  $g_s$  dependence not hidden in  $e^\phi$ .

holds for the kinetic antisymmetric tensor part in the action. The (implicit) scaling with  $g_s$  can be read off by looking at the dilaton coupling in the string frame. This will then determine the (explicit) scaling with  $g_s$  of the kinetic antisymmetric tensor part in the Einstein frame. The result is the following action in the Einstein frame:

$$S_E = \int d^D x \sqrt{g} \left\{ \frac{1}{g_s^2} \left( R - \frac{4}{D-2} (\partial\phi)^2 \right) - \frac{g_s^{2(\tilde{d}-d)/(D-2)}}{2(\tilde{d}+1)!} e^{-a\phi} F_{\tilde{d}+1}^2 \right\}. \quad (51)$$

Using the conformal transformation (50), starting from the action (51), the dual-frame action becomes

$$S_D = \int d^D x \sqrt{g} (d_p N)^{(D-2)/\tilde{d}} e^{\delta\phi} \left\{ R + \gamma (\partial\phi)^2 - \frac{1}{2(d_p N)^2 (\tilde{d}+1)!} F_{\tilde{d}+1}^2 \right\}, \quad (52)$$

where  $N$  denotes the number of stacked branes.

The dual-frame solution (29) now reads

$$\begin{aligned} ds^2 &= \alpha' \left[ \left( \frac{u}{\mathcal{R}} \right)^2 dx_d^2 + \left( \frac{\mathcal{R}}{u} \right)^2 du^2 + d\Omega_{\tilde{d}+1}^2 \right], \\ e^\phi &= g_s r_0^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]} \left( \frac{u}{\mathcal{R}} \right)^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]}. \end{aligned} \quad (53)$$

This shows that the AdS metric stays constant in  $\alpha'$  units when taking the low-energy limit if we keep the parameter  $u$  fixed. It is clear that this analysis does not hold for the conformally flat cases ( $\beta = 0$ ). We will discuss this case separately in subsection 3.3.

We want the field theory on the brane to be non-trivial and decoupled from the bulk supergravity in the limit  $\alpha' \rightarrow 0$  (low-energy limit). Non-trivial means that at least one coupling constant in the field theory should stay fixed when taking this limit. Because generically these coupling constants will be dimensionful (depending on  $\alpha'$ ), fixing this coupling constant will be non-trivial. Based on dimensional analysis and the scaling of the effective tension with  $g_s$  (see appendix B) we can deduce that the  $p$ -brane worldvolume field theory has a ('t Hooft) coupling constant  $g_f^2$  which can be written as follows:

$$g_f^2 = c_p N g_s^k (\sqrt{\alpha'})^x. \quad (54)$$

The scaling of the coupling constant with  $g_s$  is as the inverse tension  $\tau_p^{-1}$ , involving a constant  $k$  equal to

$$k = \frac{a}{2} + \frac{2d}{D-2}. \quad (55)$$

The undetermined numerical factor  $c_p$  and the parameter  $x$  depend on the specific field theory under consideration. Depending on whether  $x$  is positive or negative, the factor in (54) involving  $N$  and  $g_s$  should either become large or small in the low-energy limit in order to keep  $g_f^2$  fixed. Decoupling of the bulk supergravity is ensured when the  $D$ -dimensional Newton constant vanishes:

$$G_D = \frac{(2\pi l_s)^{(D-2)}}{32\pi^2} g_s^2 \rightarrow 0. \quad (56)$$

When using the 't Hooft coupling constant  $g_f$  as a fixed parameter, like we do, we can always keep  $g_s$  small (when  $x > 0$  we can tune  $N \rightarrow \infty$  to fix  $g_f^2$  instead of taking  $g_s \rightarrow \infty$ ) and thus in the low-energy limit  $G_D \rightarrow 0$  and ( $D$ -dimensional) gravity decouples. If we do not use

the freedom to tune  $N$ , the analysis becomes more subtle because the bulk theory can become strongly coupled ( $g_s \rightarrow \infty$ ). Sometimes this implies that the decoupling of gravity must be analysed in  $D + 1$  dimensions and in some cases this decoupling is impossible [22]. This is, for instance, the case for all domain-wall  $p$ -branes and the D6-brane in  $D = 10$ .

In general, a  $p$ -brane worldvolume field theory will involve different multiplets and thus different coupling constants. These coupling constants only differ in their dependence on  $\alpha'$ . Fixing one of these coupling constants will ensure that the other coupling constants become either small or large in the low-energy limit. When a coupling constant becomes small that sector of the  $p$ -brane field theory will decouple. For a large coupling constant what happens will depend on the  $p$ -brane field theory under consideration. When considering Yang–Mills theories with eight supercharges, the theory will involve vector- and hyper-multiplets. The vacuum structure of the theory is described by two different branches, the Higgs branch where the scalars in the hyper-multiplet have expectation values and a Coulomb branch where the scalars in the vector multiplet have expectation values. When we consider all the branes on top of each other, as we will do, the expectation values of the vector multiplet scalars (describing the relative positions of the stacked branes) are zero. This means that the theory can only be in the Higgs branch (with non-zero hyper-multiplet scalars) where all the vectors become massive (except for the  $U(1)$  centre-of-mass multiplet describing the motion of all the branes in the transverse space). The masses of the vectors will be proportional to the vector multiplet coupling constant and will therefore decouple if we take that coupling constant to infinity. We will be left with a theory consisting of scalars only. Note that we do not specify the vacuum structure of the field theory precisely. This is related to the fact that, when we consider an interpretation in terms of intersecting branes, the participating branes are *delocalized* in the compact relative transverse directions. The above sketched scenario applies when we discuss  $d$  $p$ -branes in  $D = 6$  (section 4.2) and has been discussed extensively in the case of pure  $AdS_3$  [1] and in relation with matrix models of M-theory on  $T^5$  [23, 24].

To summarize, when we speak of the field-theory limit we take<sup>†</sup>

$$\alpha' \rightarrow 0, \quad u = \text{fixed}, \quad g_f^2 = \text{fixed}. \quad (57)$$

Furthermore, from the start we assume that  $g_s \ll 1$ . This is necessary in order to make sure that the supergravity approximation is valid. As a consistency check, we must show now that the above limit takes us into the near-horizon metric. For this purpose we rewrite the harmonic function in terms of the fixed quantities  $u$  and  $g_f^2$ , leaving a power of  $\alpha'$ :

$$H = 1 + (\sqrt{\alpha'})^{(x-\tilde{d})/\beta} (g_s)^{2(k-1)/\beta} \left[ g_f^2 \left( \frac{u}{\mathcal{R}} \right)^{\tilde{d}} \left( \frac{d_p}{c_p} \right) \right]^{-1/\beta}. \quad (58)$$

Since we must have  $H \rightarrow \infty$  in the field-theory limit, it is obvious that the exponents of  $\alpha'$  and  $g_s$  should not *both* be positive. We will see in the next sections that in all interesting supersymmetric examples ( $\beta \neq 0$ ) we have

$$\frac{x - \tilde{d}}{\beta} < 0. \quad (59)$$

In some cases the power of  $g_s$  is positive,  $2(k-1)/\beta > 0$ , which could spoil the limiting behaviour of the harmonic function  $H$ . However, we can always tune  $g_s$  in such a way that we end up in the near-horizon limit when we take the field-theory limit. At the end of this section we will discuss a restriction on the parameters  $x$  and  $k$  which will be sufficient to always satisfy condition (59).

<sup>†</sup> Instead of  $\alpha' \rightarrow 0$  and  $u = \text{fixed}$  one can also say that at an energy scale  $u$  one takes the limit  $\sqrt{\alpha'} u \rightarrow 0$ .

Putting all the  $\alpha'$  dependence of the dilaton background inside the fixed quantity  $g_f^2$  we can write the dilaton background as

$$e^\phi = g_s^{1+[(D-2)a/2\Delta\beta](k-1)} (N g_s^k)^{a(D-2)(\tilde{d}-x)/4\Delta\beta x} \left[ (g_f^2)^{1/x} \left( \frac{u}{\mathcal{R}} \right) \left( \frac{d_p^{1/\tilde{d}}}{c_p^{1/x}} \right) \right]^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]}. \quad (60)$$

We can trust the background solution when the dilaton is small. For a positive sign of the overall exponent of  $g_s$  in (60), the dilaton is small everywhere except near the point  $u = 0$  or  $\infty$ , depending on the sign of  $-a(\beta + 1)/\beta$ . When the dilaton becomes large we should consider the  $S$ -dual brane. A detailed analysis of where the dilaton is large cannot be given in this general set-up. Note that in general the dilaton background depends explicitly on  $g_s$ . It turns out that this explicit dependence affects in a negative way the field-theory–supergravity dualities we would like to find. We will come back to this point at the end of this subsection after we have made some restrictions on the parameters  $k$  and  $x$ .

To trust the supergravity solution requires that the curvatures in the string frame remain small. Another way of putting this is demanding that the effective string tension (in the dual frame) times the characteristic spacetime length is large. From the metric (53) we see that the characteristic spacetime length in the dual frame is of order 1 (the length scales describing the radii of the AdS space and the sphere are of order 1 in  $\alpha'$  units). Calculating the effective string tension in the dual frame using (50) we deduce that the supergravity approximation will be valid provided

$$\tau_D = (d_p N e^{(2-k)\phi})^{2/\tilde{d}} \gg 1. \quad (61)$$

Finally, the perturbative field-theory description is valid provided the effective dimensionless coupling constant  $g_{\text{eff}}^2$ , constructed from the holographic energy scale  $u$  and  $g_f^2$ , is small:

$$g_{\text{eff}}^2 = g_f^2 u^x \ll 1. \quad (62)$$

Depending on the sign of  $x$  the perturbative field-theory description will either be valid near  $u = 0$  or  $\infty$ . Note that using the holographic energy scale  $u$  to construct the effective dimensionless coupling constant means that we use supergravity probes [11]. Also notice that this dimensionless combination of  $g_f^2$  and  $u$  is the one appearing in the dilaton expression (60).

Summarizing, to analyse supergravity–field-theory dualities we should determine the (range of) validity of the supergravity and field-theory descriptions. This leads to the following restrictions.

The supergravity solution can be trusted when

- I The string coupling is small:  $g_s = e^\phi \ll 1$  with  $e^\phi$  given in equation (60).
- II The curvatures are small:  $\tau_D = (d_p N e^{(2-k)\phi})^{2/\tilde{d}} \gg 1$  with  $\tau_D$  given by equation (61).  
The perturbative field theory is valid when
- III The effective coupling constant  $g_{\text{eff}}^2$  defined by (62) is small, i.e.  $g_{\text{eff}}^2 = g_f^2 u^x \ll 1$ .

We do not expect that we can relate supergravity to field theory in a well-defined way for generic values of the parameters  $a$ ,  $k$  and  $x$ . In the AdS case, corresponding to  $a = 0$ , the limit is well defined in the supergravity background and we can relate supergravity on the AdS background to a superconformal field theory. This case has been well studied.

Whenever  $a \neq 0$ , the dilaton background will generically spoil the ‘nice’ behaviour in the low-energy limit. With ‘nice’ we mean that there are finite regions in the supergravity



background where perturbative supergravity is valid, i.e. condition (61) is satisfied. For  $Dp$ -branes, however, we know from the work [1, 9] that things work out nicely, so a straightforward proposal would be to constrain the analysis to  $p$ -branes which behave like  $Dp$ -branes in  $D = 10$ . For  $Dp$ -branes we know that there exists another energy parameter  $U = r/\alpha'$ , related to the length of stretched strings, which can be kept fixed in the low-energy limit at the same time as the holographic energy parameter  $u$ . This implies that the relation between  $u$  and  $U$  only involves fixed quantities. In general, the relation between these two energy parameters is

$$\frac{u}{\mathcal{R}} = \alpha'^{(x+\tilde{d}-\Delta)/\Delta} g_s^{4(k-1)/\Delta} \left( \frac{d_p}{c_p} g_f^2 \right)^{-2/\Delta} U^\beta. \quad (63)$$

We propose to restrict ourselves to those  $p$ -branes for which (63) does not involve  $\alpha'$  and  $g_s$ . This constrains  $k$  and  $x$  to be equal to (whenever  $a \neq 0$ )

$$k = 1, \quad x = \Delta - \tilde{d}. \quad (64)$$

Note that the above restrictions are also satisfied for the  $a = 0$  case provided that  $\beta = 1$ . Table 5 shows that these are exactly the cases where the AdS background is embedded in a string theory. The other values of  $\beta$  ( $= \frac{1}{2}$  or  $2$ ) correspond to theories without a string coupling constant like M-theory.

The nice thing about the second formula in (64) is that it immediately tells us which coupling constant in the  $p$ -brane field theory we should keep fixed. For  $D = 10$  and  $n = 1$  we find  $x = p - 3$ , which corresponds to the scaling of the Yang–Mills coupling constant. Hence, for  $Dp$ -branes in 10 dimensions we should keep the Yang–Mills coupling constant fixed. The constraints (64) imply that condition (59) is always satisfied. They also ensure that in the dilaton background all dependence on  $g_s$  disappears:

$$e^\phi = \frac{1}{N} \left[ (g_f^2)^{1/x} \left( \frac{u}{\mathcal{R}} \right) \left( \frac{d_p^{1/\tilde{d}}}{c_p^{1/x}} \right) \right]^{-\frac{1}{8}(D-2)a[(\beta+1)/\beta]}. \quad (65)$$

This expression for the dilaton background will guarantee the existence of finite regions in the background where perturbative supergravity is valid, i.e. (61) is satisfied. In fact, in the expression (61) all  $N$  dependence will drop out. So we conclude that (64) is a *sufficient* condition on  $k$  and  $x$  for obtaining (well behaved) supergravity–field-theory dualities whenever  $a \neq 0$ . We have not proven that it is also a *necessary* condition, but we have not found any counterexamples either. As we should expect, imposing  $k = 1$  will generically relate the  $p$ -branes to intersecting  $Dp$ -branes in 10 dimensions (reduced over the relative transverse directions).

### 3.2. Pure AdS cases

The cases where the near-horizon geometry is a pure anti-de-Sitter spacetime ( $a = 0$ ) are considerably simpler, both from the supergravity point of view as well as from the field-theory point of view, since the full conformal symmetry of the background is left unbroken. The supersymmetric pure AdS cases have been listed in table 5, section 2.3.

For these cases the solution (53) reduces to

$$ds^2 = \alpha' \left[ \left( \frac{u}{\mathcal{R}} \right)^2 dx_d^2 + \left( \frac{\mathcal{R}}{u} \right)^2 du^2 + d\Omega_{\tilde{d}+1}^2 \right]. \quad (66)$$

The ('t Hooft) coupling constant  $g_f^2$  of the  $p$ -brane field theory should be independent of  $\alpha'$ , this means  $x = 0$ .

For the pure AdS cases the parameters  $\beta$  and  $\Delta$  are equal to (see section 2.3)

$$\beta = \frac{\tilde{d}}{d}, \quad \Delta = \frac{2d\tilde{d}}{d + \tilde{d}}. \quad (67)$$

The harmonic function now becomes

$$H = 1 + (l_f)^{-d} \left[ g_f^2 \left( \frac{u}{\mathcal{R}} \right)^{\tilde{d}} \left( \frac{d_p}{c_p} \right) \right]^{-1/\beta}, \quad (68)$$

where  $l_f$  is some fundamental length scale, for backgrounds in a string theory this is just  $\sqrt{\alpha'}$ , but for other theories, like M-theory, this should be  $l_p$  (the Planck length). Because  $d > 0$ , the low-energy limit  $l_f \rightarrow 0$  always takes us into the near-horizon region, defined by  $H \rightarrow \infty$ .

When embedded in a string theory, we can trust the background solution when the string coupling and the curvature are small ( $g_s \sim 0$  and  $\tau_D \gg 1$ ). Looking at (61) (taking  $k = 1$  and  $e^\phi = g_s$ ) we see that we need

$$Ng_s \gg 1, \quad (69)$$

which can only be satisfied for large  $N$ . When not discussing embeddings in string theory, we lose the parameter  $g_s$  and we only have to require small curvature, which can again only be satisfied for large  $N$ . This then immediately leads us to the (by now well-established) conjecture that large- $N$  superconformal field theory is dual to supergravity on a (pure) AdS background.

### 3.3. Conformally flat cases

In this subsection we analyse the conformally flat cases. Conformally flat cases have  $\beta = 0$ , for which the solution (53) is no longer defined. However, the singular  $\beta$  dependence can be removed by transforming the holographic energy scale  $u$  to the brane energy scale  $U$  using (63). Since this transformation requires conditions (64) to be satisfied in order for it to be independent of  $\alpha'$  and  $g_s$  we will from now on restrict ourselves to those cases. Combining this with the fact that  $\beta = 0$  yields

$$x = \tilde{d} = \frac{1}{2}\Delta = \frac{1}{4}(D - 2), \quad a = -1. \quad (70)$$

In the next section, where we discuss the  $Dp$ -branes ( $x = p - 3$ ) and ‘ $dp$ -branes’ ( $x = p - 1$ ), we will see that condition (64) is indeed satisfied for the conformally flat cases (corresponding to  $p = 5$  for  $Dp$ -branes and  $p = 2$  for ‘ $dp$ -branes’). In terms of the energy scale  $U$  the solution (53) with  $\beta = 0$  is regular:

$$ds_D^2 = \alpha' \left[ \left( g_f^2 \frac{d_p}{c_p} \right)^{-4/\Delta} dx_d^2 + \left( \frac{dU}{U} \right)^2 + d\Omega_{\tilde{d}+1}^2 \right]. \quad (71)$$

Following [9], we make the  $U \rightarrow \rho$  coordinate change

$$\rho = \ln(g_f^2 U^x) = \frac{2 \ln(g_{\text{eff}}^2)}{\Delta}, \quad (72)$$

where the dimensionless effective coupling constant has now been defined using the brane energy scale

$$g_{\text{eff}}^2 = g_f^2 U^x. \quad (73)$$

Rescaling the worldvolume coordinates by  $(g_f^2 d_p / c_p)^{2/\Delta}$  and using the dimensionless parameter  $\rho$  to rewrite the solution (71) we obtain

$$ds_D^2 = \alpha' [dx_d^2 + d\rho^2 + d\Omega_{\tilde{d}+1}^2], \quad \phi = \frac{1}{8}(D-2)\rho - \ln(\sqrt{c_p d_p} N), \quad (74)$$

so that the metric has the explicit form of a Minkowski space times a sphere:  $M_{d+1} \times S_{\tilde{d}+1}$ . Expressing the harmonic function in terms of the fixed quantities  $U$  and  $g_f$  we obtain

$$H = 1 + (\sqrt{\alpha'})^{-\Delta} g_f^2 U^{-\tilde{d}} \left( \frac{d_p}{c_p} \right), \quad (75)$$

and we see that the low-energy limit takes us into the near-horizon region and our analysis is consistent.

We can trust the supergravity background solution when the curvature and the dilaton are small. Plugging the dilaton into (61) (recall  $k = 1$  in this case) we find small curvature when  $\rho \gg 1$ . However, in that region, as long as  $\tilde{d} > 0$  we have a large dilaton, suggesting we should turn to the S-dual description. On the other hand the perturbative field-theory description is appropriate when  $\rho \ll 1$ . As long as  $\tilde{d} > 0$  we find the same behaviour for all conformally flat cases.

In the AdS cases the near-horizon ‘throat’, in the low-energy limit, closes and we end up with a near-horizon spacetime with a boundary, disconnected from the asymptotic region. This does not happen in the conformally flat cases. The near-horizon spacetime has no boundary where we can naturally place the dual-field theory and is not disconnected from the infinite throat region. Particles in this background can propagate into the throat region, as opposed to the AdS cases, where the boundary prohibits such effects (massive particles cannot even reach the boundary). This means that the throat region should be included in the dual description [8].

#### 4. Examples

In this section we will perform a more detailed investigation for some specific, supersymmetric, examples. When we have a non-trivial dilaton ( $a \neq 0$ ) our analysis will be restricted to  $p$ -branes with  $k = 1$  and  $x = \Delta - \tilde{d}$ , for reasons explained in the previous section. In subsection 4.1 we will first discuss the  $D = 10$  D $p$ -branes, including the D8-brane. In the next subsection we will discuss the d $p$ -branes in  $D = 6$  preserving only eight supersymmetries. In the next section we will focus our attention on the different black holes in  $D = 10, 6, 5, 4$  and the quantum mechanical models related to them.

##### 4.1. Ten-dimensional D $p$ -branes

The D $p$ -branes, except for the D8-brane, have already been discussed in [8, 9]. We refer to these papers for more details. The purpose of this subsection is to briefly review these papers using the notation of this paper.

All D $p$ -branes, in a low-energy limit, are described by a super-Yang–Mills theory. The Yang–Mills coupling constant for D $p$ -branes in  $D = 10$  follows from expanding the Dirac–Born–Infeld action

$$\begin{aligned} g_{YM}^2 &= \frac{1}{(2\pi\alpha')^2 \tau_p} \\ &= 2\pi g_s (2\pi l_s)^{p-3} \\ &= c_p g_s l_s^{(p-3)}, \end{aligned} \quad (76)$$

where the numerical factor  $c_p$  is equal to  $c_p = (2\pi)^{p-2}$  when considering ten-dimensional D $p$ -branes. The scaling  $x$  with  $l_s$  is given by  $x = p - 3 = \Delta - \tilde{d}$ , so the coupling constant we would like to keep fixed is the 't Hooft coupling constant  $g_f^2 = g_{YM}^2 N$ :

$$g_f^2 = c_p N g_s \sqrt{\alpha'}^{p-3}. \quad (77)$$

In the low-energy limit the scalars in the Yang–Mills theory will decouple. Other parameters that we need are

$$\Delta = 4, \quad \tilde{d} = 7 - p, \quad \beta = \frac{1}{2}(5 - p), \quad a = \frac{1}{2}(3 - p). \quad (78)$$

As was shown in a more general context ( $k = 1$  and  $x = \Delta - \tilde{d}$ ), the constraint (59) is satisfied and we are able to fix two energy quantities at the same time. The specific relation between the two energy scales  $U$  and  $u$  for D $p$ -branes is given by

$$U^{5-p} = g_f^2 \left( \frac{u}{\mathcal{R}} \right)^2 \left( \frac{d_p}{c_p} \right). \quad (79)$$

We can freely choose between them corresponding to considering holographic supergravity probes or D $p$ -brane probes. The explicit values of the constants  $d_p$  and  $c_p$  can be found in the appendix.

To keep  $g_f^2$  fixed for  $p < 3$  we do not need to tune  $N$ , because we always take  $g_s$  to be small (for the supergravity solution to be valid we need low energy and small coupling). For  $p > 3$  we have to take  $N$  to infinity to keep  $g_f^2$  fixed. As is well known, in the special case that  $p = 3$  we lose the non-trivial dilaton and obtain a pure  $AdS_5 \times S_5$  background. The case of  $p = 5$  is special because it is conformally flat. This case was already discussed in subsection 3.3. We will not discuss the case  $p = 7$  corresponding to  $\tilde{d} = 0$ .

Plugging in the different parameters the dilaton background (60) takes the form

$$e^\phi = \frac{1}{N} \left[ (g_f^2)^{1/(p-3)} \left( \frac{u}{\mathcal{R}} \right) \left( \frac{d_p^{1/(7-p)}}{c_p^{1/(p-3)}} \right) \right]^{(p-3)(7-p)/2(5-p)}. \quad (80)$$

We see that by making  $N$  large, we can always make the dilaton small, except when we approach the points  $u = 0$  or  $\infty$ , depending on  $p$ . This analysis coincides with [9] and we refer to that paper for a more detailed discussion of all D $p$ -branes with  $p < 8$ . We have summarized some of the results in table 6.

We would now like to focus our attention on the only D $p$ -brane missing in previous discussions, the D8-brane.

We find for the D8-dilaton background

$$e^\phi = \frac{1}{N} \left[ g_f^2 \left( \frac{u}{\mathcal{R}} \right)^5 \left( \frac{16}{\pi} \right) \right]^{1/6}, \quad (81)$$

which means that the dilaton background is small everywhere as long as we take large  $N$ . This is necessary anyway since, in order to keep  $g_f^2$  for the D8-brane fixed, we need  $N \rightarrow \infty$ . This means that we consider the 't Hooft limit of the (non-renormalizable) field theory. Only when we approach  $u = \infty$  does the dilaton grows and we have to consider the S-dual situation which would involve M9-branes in M-theory. We would like to point out that in the limit of  $N \rightarrow \infty$ , the size of the region where the dilaton is large can be made as small as a point. In the present case of the D8-brane  $u = \infty$  is the actual position of the D8-domain wall, as opposed to branes with  $p < 6$ , where  $u = 0$  denotes the position of the brane.

For the dimensionless effective coupling (62) and the string tension (61) we find

$$g_{\text{eff}}^2 = g_f^2 u^5, \quad \tau_D = \left( \frac{1}{4\pi} \left[ g_f^2 \left( \frac{u}{\mathcal{R}} \right)^5 \left( \frac{16}{\pi} \right) \right]^{1/6} \right)^{-2}, \quad (82)$$

so that supergravity and perturbative Yang–Mills are valid in the same region for the D8-brane, near  $u = 0$ :

$$\begin{aligned} u^5 \ll 1/g_f^2 &\rightarrow \text{FT}, \\ u^5 \ll 1/g_f^2 &\rightarrow \text{SUGRA}. \end{aligned} \quad (83)$$

This also happens for the D6-brane (although in that case we need to go to the strong-coupling background which involves the  $D = 11$  KK-monopole, and in  $D = 11$  gravity does not decouple) and would suggest a contradiction in the sense that we have two, apparently different, weakly-coupled descriptions. One reason could be that the decoupling of gravity of the field theory in these cases is rather subtle [8, 9]. However, as we discussed earlier, when fixing the 't Hooft coupling constant, the decoupling of gravity in the 't Hooft limit of the field theory seems guaranteed for all branes. One reason not to trust the decoupling of gravity is the fact that  $g_s N$  is the open string coupling constant, which becomes infinite in this case. Two open strings can join to form closed strings (which includes gravitons). It is hard to understand why the strongly-coupled open string theory would not produce these closed strings (and thus decouple from gravity) in the low-energy limit<sup>†</sup>. For now, however, we conjecture that  $DW_{10}$  supergravity near  $u = 0$  equals  $D = 9$  perturbative SYM theory in the 't Hooft limit. Another proposal for the interpretation of this result, although not unrelated to the 'decoupling of gravity' argument, can be found in [11]. In any case, it is interesting to note that the D6- and D8-branes behave in similar ways, a point of view that is also advocated from considering the embedding of these branes in M-theory (where they both develop an extra compact direction) [15].

We end this subsection with table 6 in which we summarize the results for ten-dimensional  $Dp$ -branes.

**Table 6.** This table shows for which values of  $u$ , in the near-horizon geometry, the restrictions I, II and III of subsection 3.1, i.e.  $e^\phi \ll 1$ ,  $\tau_D \gg 1$  and  $g_{\text{eff}} \gg 1$ , are satisfied for the case of  $Dp$ -branes in  $D = 10$  dimensions. The bottom row indicates the behaviour of  $N$  (see also the discussion in the text).

$Dp$	0, 1, 2	3	4	6	8
$g_{\text{eff}} \ll 1$	$\infty$	$\forall$	0	0	0
$\tau_D \gg 1$	0	$\forall$	$\infty$	0	0
$e^\phi \gg 1$	0	$\emptyset$	$\infty$	0	$\infty$
$N$	$\forall$	$\gg 1$	$\infty$	$\infty$	$\infty$

#### 4.2. Six-dimensional $dp$ -branes

Using the constraint  $x = \Delta - \tilde{d} = p - 1$ , the relevant dynamics should be governed by scalars. Because these  $dp$ -branes are related to intersecting  $Dp$ -branes the worldvolume theory should again contain supersymmetric Yang–Mills. However, the Yang–Mills coupling constant in this case is not kept fixed but instead diverges. As we mentioned in subsection 3.1, when we consider the  $dp$ -branes on top of each other, we are in the Higgs branch vacuum of the theory.

<sup>†</sup> We thank K Skenderis for a discussion on this point.

In the limit of diverging Yang–Mills coupling constant all the vectors decouple and we are left with a scalar theory. The (scalar) coupling constant we keep fixed is

$$g_f^2 = c_p N g_s \sqrt{\alpha'}^{p-1}. \quad (84)$$

Other parameters we need are

$$\Delta = 2, \quad \tilde{d} = 3 - p, \quad \beta = 2 - p, \quad a = (1 - p). \quad (85)$$

The quantities  $u$  and  $U$  can both be fixed at the same time. In this specific case we obtain

$$U^{2-p} = g_f^2 \left( \frac{u}{\mathcal{R}} \right) \left( \frac{d_p}{c_p} \right). \quad (86)$$

We see that until now the analysis resembles quite accurately that of the  $D = 10$  D $p$ -branes.

In the following we will ignore the cases  $p = 1$  (no dilaton),  $p = 2$  (conformally flat, see subsection 3.3) and  $p = 3$  ( $\tilde{d} = 0$ ). To keep  $g_f^2$  fixed for  $p = 0$  we find that it is possible but not necessary for  $N$  to go to infinity. For  $p = 4$  we have to take  $N$  to infinity to keep  $g_f^2$  fixed.

Substituting the different parameters the dilaton background (60) becomes

$$e^\phi = \frac{1}{N} \left[ (g_f^2)^{1/(p-1)} \left( \frac{u}{\mathcal{R}} \right) \left( \frac{d_p^{1/(3-p)}}{c_p^{1/(p-1)}} \right) \right]^{(p-1)(3-p)/2(2-p)}. \quad (87)$$

Focusing on the d0- and d4-brane, we next analyse where the field-theory and supergravity description are valid using (62) and (61). For the d0-brane we find that the perturbative field-theory description is valid near  $u = \infty$  and the (S-dual) supergravity description is valid near  $u = 0$ :

$$\begin{aligned} u \gg g_f^2 &\rightarrow \text{FT}, \\ u \ll g_f^2 &\rightarrow \text{SUGRA}. \end{aligned} \quad (88)$$

This is the typical behaviour in field-theory–supergravity dualities, both descriptions being valid in different regions, avoiding contradictions. For the d0-brane it mimics the ten-dimensional D0-brane situation. As is clear from the above we should turn to the S-dual background in the infrared. However, what is the S-dual background? We know that IIA superstring theory on a  $K3$  manifold is S-dual to heterotic superstring theory on a  $T^4$ . The S-dual solution is nothing but the 0-brane of heterotic string theory on  $T^4$ , where the extreme black hole is charged with respect to a Kaluza–Klein gauge field or with respect to a  $U(1)$  subgroup of the  $SO(32)$  gauge fields. This solution is fundamental in the sense that its mass is independent of  $g_s$  (like the fundamental string) and it has a curvature singularity at  $u = 0$ . This curvature singularity should be resolved by the ‘strong-coupling limit’ of the corresponding quantum mechanics on the d0-branes. In this respect this situation resembles the D1–F1 situation in IIB superstring theory in  $D = 10$  [8]. It would be interesting to determine explicitly the infrared limit of the corresponding quantum mechanics model. We will say more about this case in the next section when we discuss 0-branes in various dimensions.

For the ‘d4-brane’ the situation is similar to that of the D8 domain wall in  $D = 10$ . We find that the field-theory and supergravity description are valid in the same region near  $u = 0$  apparently leading to a contradiction:

$$\begin{aligned} u^3 \ll 1/g_f^2 &\rightarrow \text{FT}, \\ u^3 \ll 1/g_f^2 &\rightarrow \text{SUGRA}. \end{aligned} \quad (89)$$

As was explained in the previous subsection, we have no good explanation for this puzzling situation. Our results suggest the equivalence between  $DW_6$  supergravity near  $u = 0$  and a  $D = 5$  perturbative (scalar) field theory, which is the Higgs branch of a  $D = 5$  SYM theory with eight supercharges, in the 't Hooft limit ( $N \rightarrow \infty$ ). For arguments in favour of another interpretation of this result we refer to [11]. Near  $u = \infty$  we should use the S-dual background, which should be embedded in heterotic string theory on  $T^4$ . This involves the  $D = 10$  heterotic solitonic 5-brane with one leg of the 5-brane wrapped around the  $T^4$ . Note that the  $D = 10$  interpretation of the d4-brane is a delocalized D4–D8 brane solution of type IIA string theory. Duality conjectures based upon a localized D4–D8-brane system embedded in type I' string theory have been discussed in [25, 26].

This finishes our discussion on  $dp$ -branes preserving eight supersymmetries. All features of  $D = 10$   $Dp$ -branes reappear in the  $D = 6$  context. In table 7 we have summarized our results on the six-dimensional ‘ $dp$ -branes’.

**Table 7.** This table shows for which values of  $u$ , in the near-horizon geometry, the restrictions I, II and III of subsection 3.1, i.e.  $e^\phi \ll 1$ ,  $\tau_D \gg 1$  and  $g_{\text{eff}} \gg 1$ , are satisfied for the case of ‘ $dp$ -branes’ in  $D = 6$  dimensions. The bottom row indicates the behaviour of  $N$  (see also the discussion in the text).

$dp$	0	1	4
$g_{\text{eff}} \ll 1$	$\infty$	$\forall$	0
$\tau_D \gg 1$	0	$\forall$	0
$e^\phi \gg 1$	0	$\emptyset$	$\infty$
$N$	$\forall$	$\gg 1$	$\infty$

## 5. Quantum mechanics and 0-branes

In this section we will consider the special case of 0-branes in various dimensions. In particular, we will construct the Hamiltonian of the (boundary) quantum mechanics. We expect the Hamiltonian to be invariant under (generalized) conformal transformations because of the (conjectured) duality between the supergravity theory and the boundary theory. Because the metric is pure AdS, the isometries of the metric will induce the conformal invariance in the boundary QM. It is clear that a non-trivial dilaton breaks the conformal isometries of the background. As will be explained we can only expect to find generalized conformal invariance [27] in those cases.

Models of (super-)conformal quantum mechanics have been studied in [28, 29]. The relation of these models with black holes was pointed out recently in [30]. The exact map between  $D = 2$  (pure) AdS (quantum) supergravity and the quantum mechanics model at the boundary, requires a special discussion and is not yet well understood [2, 31–33].

We will first give a general discussion, applicable to any dimension. Next we will focus on the special supersymmetric cases in  $D = 10$  (the D0-brane),  $D = 6$  (the d0-brane, already discussed partially in the previous section) and the pure AdS cases in  $D = 5$  (the m0-brane) and  $D = 4$  (the Reissner–Nordström (RN) extreme black hole). The RN black hole and the D0-brane have already been discussed in [30] and [34], respectively. We will show that these cases fit naturally into our general analysis.

To construct the quantum mechanics Hamiltonian we consider a charged probe particle in the (supersymmetric) near-horizon background of  $N$  stacked 0-branes. Whenever we have a non-trivial dilaton we will restrict ourselves to those cases for which conditions (64) which for  $p = 0$  read  $k = 1$  and  $x = \Delta - D + 3$ . The (bosonic) Lagrangian for a particle with mass

$m$  and charge  $q$ , moving in the dual-frame (near-horizon) background of  $N$  stacked 0-branes then reads

$$\mathcal{L} = m L g_s^{-1/(D-3)} e^{-[(D-4)/(D-3)]\phi} \sqrt{|\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}^{(D)}|} + q A_\mu \dot{x}^\mu, \quad (90)$$

where the dot represents derivatives with respect to the worldline time and  $L$  is the (dimensionless) parameter defined in appendix A (see also section 3). The  $D$ -dimensional 0-brane solution in the dual-frame metric  $g_{\mu\nu}^{(D)}$  is given by the expression (53), taken for  $p = 0$ . This expression is constant in  $\alpha'$  units. Introducing the canonical momentum  $P_\mu = \partial\mathcal{L}/\partial\dot{x}^\mu$  we can write down the mass-shell constraint for the probe particle which is

$$(P_\mu - q A_\mu)(P_\nu - q A_\nu) g_D^{\mu\nu} = m^2 L^2 g_s^{-2/(D-3)} e^{-2[(D-4)/(D-3)]\phi}. \quad (91)$$

We would like to solve this equation for  $P_t = -\mathcal{H}$ . Substituting the background metric (53) into the mass-shell equation (91) and using the fact that in the static gauge  $A_\mu \dot{x}^\mu = A_t$ , we solve for  $P_t$  and find the Hamiltonian

$$\mathcal{H} = \frac{u}{\mathcal{R}} \sqrt{\left(\frac{u}{\mathcal{R}}\right)^2 P_u^2 + \tilde{L}^2 + \alpha' m^2 L^2 g_s^{-2/(D-3)} e^{-2[(D-4)/(D-3)]\phi} - q A_t}, \quad (92)$$

where  $\tilde{L}^2 \equiv P_i P_j g^{ij} \alpha'$ , the (dimensionless) squared angular momentum vector over the sphere, not to be confused with  $L$  which is just a constant. Using the identity  $A - B = (A^2 - B^2)/(A + B)$  we can write this Hamiltonian as

$$\mathcal{H} = \frac{P_u^2}{2f} + \frac{g}{2f}, \quad (93)$$

with  $f$  and  $g$  given by

$$\begin{aligned} f &= \frac{1}{2} e^{-[(D-4)/(D-3)]\phi} \left(\frac{\mathcal{R}}{u}\right)^3 \left\{ \sqrt{\alpha' m^2 (d_p N)^{2/(D-3)} + e^{2[(D-4)/(D-3)]\phi} \left(\frac{u}{\mathcal{R}}\right)^2 \left[ P_u^2 + \mathcal{R}^2 \frac{\tilde{L}^2}{u^2} \right]} \right. \\ &\quad \left. + \left(\frac{\mathcal{R}}{u}\right) e^{[(D-4)/(D-3)]\phi} A_t q \right\}, \\ g &= e^{-2[(D-4)/(D-3)]\phi} \left(\frac{\mathcal{R}}{u}\right)^2 \left\{ \alpha' m^2 (d_p N)^{2/(D-3)} \right. \\ &\quad \left. - \left(\frac{\mathcal{R}}{u}\right)^2 e^{2[(D-4)/(D-3)]\phi} A_t^2 q^2 + e^{2[(D-4)/(D-3)]\phi} \tilde{L}^2 \right\}. \end{aligned} \quad (94)$$

We would like to substitute the expressions for the different background fields and see whether we can simplify the expressions for  $f$  and  $g$ . Note that we use the electric potential instead of the dual magnetic potential we are using in the rest of the paper. The solution for the (electric) vector potential is

$$A_t = (g_s H)^{-1}. \quad (95)$$

The expression for the dilaton is given in (53), but we restrict ourselves to cases with  $k = 1$  and  $x = \Delta - D + 3$  so we can use the dilaton expression in (65). The reader may now check that for all supersymmetric 0-branes in the near-horizon limit, the following identity holds:

$$\frac{\mathcal{R}}{u} A_t e^{[(D-4)/(D-3)]\phi} = (d_p N)^{1/(D-3)} \sqrt{\alpha'}, \quad (96)$$



This identity enables us to simplify the expressions for  $f$  and  $g$  considerably. Introducing a function  $\mathcal{V}(u)$  which is equal to

$$\mathcal{V}(u) = (d_p N)^{2/(D-3)} e^{-2[(D-4)/(D-3)]\phi} \left(\frac{\mathcal{R}}{u}\right)^2, \quad (97)$$

we can write the functions  $f$  and  $g$  as follows:

$$f = \frac{1}{2} \sqrt{\mathcal{V}(u)} \left(\frac{\mathcal{R}}{u}\right)^2 \left\{ \sqrt{\alpha' m^2 + \frac{1}{\mathcal{V}(u)} \left[ P_u^2 + \mathcal{R}^2 \frac{\tilde{L}^2}{u^2} \right]} + q \sqrt{\alpha'} \right\}, \quad (98)$$

$$g = \mathcal{V}(u) \alpha' (m^2 - q^2) + \mathcal{R}^2 \frac{\tilde{L}^2}{u^2}.$$

From this form of  $f$  and  $g$  it is clear that in the field-theory limit ( $\alpha' \rightarrow 0$ ) the quantum mechanics model is well defined because  $m$  and  $q$  have an (implicit)  $\alpha'$  dependence ( $m, q \sim 1/\sqrt{\alpha'}$ ) cancelling all  $\alpha'$ 's appearing in  $f$  and  $g$ . We also see that in the extreme limit ( $m^2 = q^2$ ) the function  $g$  simplifies considerably:  $g = \mathcal{R}^2 \tilde{L}^2 / u^2$ . This is the centrifugal potential for a free particle. Except for a ‘conformal’ factor in  $f$  (depending on  $u$ ) and the somewhat unusual  $u$  dependence of  $\mathcal{V}(u)$ , this should be a Hamiltonian of (‘relativistic’) conformal quantum mechanics as described in [30] as long as we keep the dilaton term in (97) fixed under conformal transformations. The appearance of the  $u$ -dependent ‘conformal’ factor (and the particular  $u$  dependence of  $\mathcal{V}(u)$ ) is the result of our choice of phase space canonical variables ( $u, P_u$ ) and can be changed by making another choice of canonical variables. Because  $\mathcal{V}(u)$  is just a power of  $u$ , when we take  $u' \propto \mathcal{V}(u)^{1/4}$  and rewrite the Hamiltonian using the canonical variables  $u'$  and  $P_{u'}$ , we remove the position dependence in this ‘conformal’ factor in  $f$ . The ‘non-relativistic’ limit [30] is one in which the function  $\mathcal{V}(u)$  goes to infinity and the expression  $\mathcal{V}(u) \alpha' (m^2 - q^2)$  is kept fixed. In that limit, except for an infinite ‘conformal’ factor which can be removed by using another canonical set of variables, we obtain the conformal quantum mechanics of [28].

The generators of special conformal and dilatation transformations are equal to

$$\mathcal{K} = f u^2, \quad \mathcal{D} = u P_u. \quad (99)$$

Together with the generator of time translations, the Hamiltonian  $\mathcal{H}$ , these generators satisfy the following  $SL(2, \mathbb{R})$  algebra [29]

$$[\mathcal{H}, \mathcal{D}] = \mathcal{H}, \quad [\mathcal{D}, \mathcal{K}] = \mathcal{K}, \quad [\mathcal{H}, \mathcal{K}] = \mathcal{D}. \quad (100)$$

This is only true when the effective coupling constant  $g_{\text{eff}}^2$  of the QM model under consideration is fixed under scale transformations. Whenever we have a non-trivial dilaton the QM model will not be conformal invariant by itself. Only when introducing a transformation of  $g_f^2$  to keep  $g_{\text{eff}}^2$  fixed under conformal transformations, will the QM model be invariant under what are called generalized conformal transformations [27].

We will now discuss as special cases the supersymmetric black holes in  $D = 10, 6, 5, 4$ .

- *The  $D = 10$  D0-brane.* For  $D = 10$  the function  $\mathcal{V}(u)$  is given by

$$\mathcal{V}(u) = (N d_p)^{4/5} \left( \frac{N c_p}{\mathcal{R}^3 g_{\text{eff}}^2} \right)^{6/5} \left( \frac{\mathcal{R}}{u} \right)^2. \quad (101)$$

Remember that there is some  $u$  dependence hidden in  $g_{\text{eff}}^2$  and that the model is invariant under generalized conformal transformations keeping  $g_{\text{eff}}^2$  fixed. The expression differs

from the one given in [34]. Performing a coordinate transformation, going from the variables  $u$  and  $P_u$  to the canonical variables  $U$  and  $P_U$  (using (63)) we lose the position dependence of the ‘conformal’ factor in  $f$  and we obtain the result of [34]. We refer to that paper and [33] for more details on the relation of this model with (M)atrix theory.

- *The  $D = 6$  d0-brane.* For  $D = 6$  the function  $\mathcal{V}(u)$  is given by

$$\mathcal{V}(u) = Nd_p \left( \frac{Nc_p}{\mathcal{R}g_{\text{eff}}^2} \right) \left( \frac{\mathcal{R}}{u} \right)^2. \quad (102)$$

As in the previous example, there is hidden  $u$  dependence in  $g_{\text{eff}}^2$  and the model is therefore invariant under generalized conformal transformations keeping  $g_{\text{eff}}^2$  fixed. The canonical set of variables  $(r, P_r)$  will remove the position dependence of the ‘conformal’ factor in  $f$  if  $u \propto r^{-4}$  (this means we cannot use  $U$ , which is proportional to  $\sqrt{u}$ , as we did in the D0-brane case). Considering the extremal case and purely radial motion ( $\vec{L}^2 = 0$ ), the large- $u$  (small- $\mathcal{V}$ ) limit should be considered ‘ultra-relativistic’ and the small- $u$  (large- $\mathcal{V}$ ) limit should be considered a ‘classical’ limit of the quantum mechanics model in the sense that the correction terms to  $m$  in the function  $f$  can be neglected. This suggests that the singularity in the heterotic 0-brane solution (see subsection 4.2) is resolved by a (‘classical’) free conformal quantum mechanics model. This would be in correspondence with the resolution of the singularity in the fundamental string solution by a free-orbifold conformal field theory [38].

- *The  $D = 5$  m0-brane.* For  $D = 5$  the function  $\mathcal{V}(u)$  is given by

$$\mathcal{V}(u) = Nd_p \left( \frac{\mathcal{R}}{u} \right)^2. \quad (103)$$

The main difference with the previous cases is the fact that we no longer have a hidden  $u$  dependence in some effective coupling constant  $g_{\text{eff}}^2$ . This QM model is conformal invariant without having to introduce an extra scaling law for  $g_f^2$  which is a consequence of the fact that the background is pure AdS.

- *The  $D = 4$  extreme RN black hole.* For  $D = 4$  the function  $\mathcal{V}(u)$  is given by

$$\mathcal{V}(u) = (Nd_p)^2 \left( \frac{\mathcal{R}}{u} \right)^2, \quad (104)$$

which is the same as the expression for the m0-brane, except for a factor of  $Nd_p$ . Again this is not the same as the expression in [30], because of the different phase space variables we are using.

So far, we have been considering the conformal quantum mechanics model of a single 0-brane. It has been argued recently [35] that the quantum mechanical model governing the fluctuations of  $N$  stacked 0-branes is given by the  $N$ -particle Calogero model. This is based on the observation that the  $N$ -particle Calogero model is related, in the large- $N$  limit, to the reduction of a  $D = 2$  Yang–Mills theory describing the fluctuations of  $N$  stacked 1-branes.

The expression of the  $N$ -particle Calogero Hamiltonian, acting on the subspace of totally symmetric wavefunctions, is given by

$$\mathcal{H} = \frac{1}{2} \sum_i p_i^2 + \sum_{i < j} \frac{\ell(\ell - 1)}{(q_i - q_j)^2}. \quad (105)$$

Here  $\ell$  is a constant and  $(q_i, p_i)$  ( $i = 1, \dots, N$ ) are a set of  $2N$  canonical variables. In terms of the ‘coupled’ momentum operators

$$\pi_i = p_i + \ell \sum_{i \neq j} \frac{K_{ij}}{q_i - q_j}, \quad (106)$$

where  $K_{ij}$  are the so-called exchange operators, the Calogero Hamiltonian can be brought into the free form [36]

$$\mathcal{H} = \sum_i \pi_i^2. \quad (107)$$

In terms of annihilation and creation operators  $(a_i, a_i^\dagger)$  satisfying a so-called  $S_N$ -extended Heisenberg algebra the Hamiltonian can be written as [37]

$$\mathcal{H} = \frac{1}{2} \sum_i \{a_i, a_i^\dagger\}. \quad (108)$$

In terms of these operators the generators of the  $SL(2, \mathbb{R})$  algebra (100), which is called the ‘vertical’  $SL(2, \mathbb{R})$  algebra, are given by the Calogero Hamiltonian  $\mathcal{H}$  and the two operators

$$B_2^+ = \frac{1}{2} \sum_i (a_i^\dagger)^2, \quad B_2^- = \frac{1}{2} \sum_i (a_i)^2. \quad (109)$$

The point we want to make here is that the Calogero Hamiltonian is also part of another so-called ‘horizontal’  $SL(2, \mathbb{R})$  algebra with generators  $\{L_{-1}, L_0, L_1\}$  such that  $L_0 - \mathcal{H}$  is a pure constant and the other two generators are given by

$$\begin{aligned} L_1 &= \sum_i a_i, \\ L_{-1} &= \sum_i [\alpha (a_i^\dagger)^2 a_i + (1 - \alpha) a_i (a_i^\dagger)^2] + 2(\lambda - \frac{1}{2}) \sum_i a_i^\dagger, \end{aligned} \quad (110)$$

with  $\alpha$  and  $\lambda$  arbitrary constants. It turns out that this ‘horizontal’  $SL(2, \mathbb{R})$  algebra can naturally be extended to a Virasoro algebra which acts as a kind of spectrum-generating algebra of the Calogero model. For more details, see [39]<sup>†</sup>. It would be interesting to see whether this hidden Virasoro symmetry will play a role in the microscopic description of black holes in terms of the Calogero model.

## 6. The type I/heterotic 6-brane

In this section we will discuss some properties of the type I/heterotic 6-brane solution we discussed in subsection 2.3 as an example of a brane whose near-horizon geometry is a conformally flat space. We will only discuss some classical aspects of the solution and not the field-theory limit so that this section can be read independently of the previous sections.

The type I/heterotic 6-brane is magnetically charged under an  $U(1)$  subgroup of the 10D Yang–Mills gauge field. The complete solution reads

$$\begin{aligned} ds_6^2 &= -dt^2 + dy_1^2 + \dots + dy_6^2 + H^2 d\vec{x} d\vec{x}, \\ F_{mn} &= \epsilon_{mnp} \partial_p H, \quad e^{2\phi} = H, \end{aligned} \quad (111)$$

<sup>†</sup> Recently, it has been shown that any scale-invariant mechanics of one variable has the symmetries of a full Virasoro algebra [40].

with  $m, n = 1, 2, 3$  and  $H$  a harmonic function in the transverse coordinates  $\vec{x} = (x_1, x_2, x_3)$ . The first thing to notice is that this solution is not supersymmetric in 10 dimensions. Namely, the heterotic susy variations of the gravitino  $\psi_M$ , the dilatino  $\lambda$  and the gaugino  $\chi$  are

$$\begin{aligned}\delta\psi_M &= \left[\partial_M + \frac{1}{4}\Omega_M^{(-)AB}\Gamma_{AB}\right]\epsilon, \\ \delta\lambda &= \left[-\Gamma^M\partial_M\phi + \frac{1}{2\cdot 3!}\Gamma^{MNP}H_{MNP}\right]\epsilon, \\ \delta\chi &= \Gamma^{MN}F_{MN}\epsilon,\end{aligned}\tag{112}$$

where  $\Omega_M^{(-)AB}$  is defined as a combination of the spin connection and the torsion ( $A, B$  are flat indices):

$$\Omega_M^{(-)AB} \equiv \omega_M^{AB} - H_M^{AB}.\tag{113}$$

Since the torsion is trivial for the type I/heterotic 6-brane solution, none of these variations can vanish (assuming that the Killing spinor does not depend on the worldvolume directions). So how is this solution related to the other *supersymmetric* branes? In heterotic string theory we have only the NS5-brane and the KK-monopole which give rise to magnetic charges and the bound state of both is given by

$$\begin{aligned}ds_{\bar{5}\times KK}^2 &= -dt^2 + dy_1^2 + \dots + dy_5^2 + H_1 \left[ \frac{1}{H_2} (dx_4 + V_m dx^m)^2 + H_2 d\vec{x} d\vec{x} \right], \\ H_{4mn} &= \epsilon_{mnp}\partial_p H_1, \quad \partial_m H_2 = \epsilon_{mnp}\partial_n V_p, \quad e^{2\phi} = H_1.\end{aligned}\tag{114}$$

Obviously, in order to obtain the 6-brane we must identify the two harmonics:  $H_1 = H_2$ . However, upon reduction to nine dimensions the gauge fields coming from this bound state enter only the right-moving sector (see, e.g., [41]), but the gauge field coming from the 6-brane is part of the left-moving sector. Therefore, there cannot be any heterotic  $O(2, 1)$ -duality that relates both to each other. Instead, in order to find a correspondence we have to make sure that the nine-dimensional gauge fields are in the same sector, which can be done by changing the sign of the torsion, i.e. we replace (114) by

$$\begin{aligned}ds_{\bar{5}\times KK}^2 &= -dt^2 + dy_1^2 + \dots + dy_5^2 + [(dx_4 + V_m dx^m)^2 + H^2 d\vec{x} d\vec{x}], \\ H_{4mn} &= -\epsilon_{mnp}\partial_p H, \quad \partial_m H = \epsilon_{mnp}\partial_n V_p, \quad e^{2\phi} = H,\end{aligned}\tag{115}$$

where the subscript  $\bar{5} \times KK$  indicates that we consider a bound state of an anti-5-brane and a KK-monopole. The equations of motions are invariant under this sign change, but it breaks supersymmetry! After reducing over  $x_4$  the nine-dimensional gauge fields have now the same chirality and can be rotated into each other. In fact the 6-brane is related to (115) by the  $O(2, 1)$  transformation

$$e^{-(1/\sqrt{2})J_-} e^{\sqrt{2}J_+} = \begin{pmatrix} 1 & 1 & -\sqrt{2} \\ 1 & 1 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{pmatrix},\tag{116}$$

where  $J_{\pm}$  are the boost generators of  $O(2, 1)$  (we use the notation of [42]) and this transformation relates the nine-dimensional gauge fields to each other

$$\begin{pmatrix} V_\mu \\ -V_\mu \\ 0 \end{pmatrix}_{\bar{5}\times KK} \leftrightarrow \begin{pmatrix} 0 \\ 0 \\ \sqrt{2} V_\mu \end{pmatrix}_6.\tag{117}$$

There is no Wilson line created by this duality, i.e.  $V_x = 0$ . The dilaton and the compactification radius ( $R^2 = g_{44}$ ) are related to each other by

$$e^{2\phi_6} = \frac{1}{2} e^{2\phi_{\bar{5} \times K K}}, \quad R_6 = \frac{1}{2} R_{\bar{5} \times K K} = \frac{1}{2}, \quad (118)$$

i.e. the string coupling constant as well as the compactification radius are only half of the original values. Thus, the 6-brane can be seen as a bound state of an anti-5-brane and a KK-monopole at the self-dual radius (defined by  $H_1 = H_2$  giving  $R_{\bar{5} \times K K} = 1$ ), as discussed in [43]. Related examples of non-supersymmetric brane solutions have been discussed in [44, 45].

It is easy to construct intersections of 6-branes. Since the worldvolume directions trivially factorizes for the 6-brane, it is straightforward to combine up to three 6-branes

$$\begin{aligned} ds_{6 \times 6 \times 6}^2 &= -dt^2 + H_1^2 d\vec{x}_1 d\vec{x}_1 + H_2^2 d\vec{x}_2 d\vec{x}_2 + H_3^2 d\vec{x}_3 d\vec{x}_3, \\ F_{mn}^i &= \epsilon_{mnp} \partial_p H_i, \quad e^{2\phi} = H_1 H_2 H_3, \end{aligned} \quad (119)$$

where  $\vec{x}_i$  are the coordinates of three three-dimensional subspaces ( $i = 1, \dots, 3$ ). We see, that two 6-branes intersect over a 3-brane (common worldvolume) and all three intersect over a point. Introducing polar coordinates the harmonic functions can be written as

$$H_i = 1 + \frac{q_i}{r_i}. \quad (120)$$

Taking the limit where we can ignore the constant parts in  $H_i$  and introducing  $r_i = q_i e^{q_i y^i}$  the metric and dilaton becomes

$$ds_{6 \times 6 \times 6}^2 = -dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + q_i^2 d\Omega_2^{(i)}, \quad e^{2\phi} = e^{q_i y^i}. \quad (121)$$

Hence, we obtain a linear dilaton and the geometry is  $M_4 \times S_2 \times S_2 \times S_2$  where  $M_4$  is a flat four-dimensional Minkowski spacetime.

The type I/heterotic 6-brane also has an electric dual, which can be constructed by reducing the 6-brane to four dimensions and applying the standard rules of S-duality and a subsequent oxidation to 10 dimensions. As a result one finds the following type I/heterotic 0-brane solution:

$$\begin{aligned} ds_0^2 &= -\frac{1}{H^2} dt^2 + dy_1^2 + \dots + dy_9^2, \\ A_0 &= 1/H, \quad e^{-2\phi} = H. \end{aligned} \quad (122)$$

In analogy to the 6-brane this type I/heterotic 0-brane can be obtained via an  $O(2, 1)$  T-duality from an anti-string-wave bound state at the self-dual radius. Using the  $O(2, 1)$  transformation it is straightforward to construct a bound state of a 6- and a 0-brane which is given by

$$ds^2 = -\frac{1}{H_0^2} dt^2 + H_1^2 d\vec{x} d\vec{x} + dy_1^2 + \dots + dy_6^2. \quad (123)$$

This solution can immediately be verified by compactifying all  $y$  coordinates which yields the standard dyonic Reissner–Nordström black hole.

## 7. Conclusions

In this paper, we presented a systematic overview of the near-horizon geometry of any two-block  $p$ -brane solution and discussed the corresponding domain-wall/QFT duality, thereby generalizing the discussion of [8, 9]. The near-horizon geometry of the general  $p$ -brane solution is singular, but the singularity is only contained in a conformal factor; for all  $p$ -branes the Weyl-tensor is regular. In fact, in a conformally rescaled so-called ‘dual’ frame, the

spacetime factorizes into an anti-de Sitter space and a sphere [5, 6]. The analysis of appendix B shows, that this ‘dual-frame’ metric coincides with the sigma-model metric [49], which enters the Nambu–Goto action of a dual-brane probe [5]. It is for this reason that the metric is called a ‘dual-frame’ metric.

By reducing over the spherical part of the  $p$ -brane near-horizon geometry one obtains a domain-wall solution. For any  $p$ -brane we established the explicit form of this domain wall in terms of the parameters of the original  $p$ -brane solution. There are two special cases. First, the domain wall can become an AdS space, which corresponds to the well-known regular branes with a trivial dilaton. Secondly, the domain wall can become flat. In 10 dimensions this is the case for the 5-branes and for the heterotic/type I 6-brane. The latter case is new and was discussed in section 6. In particular, we showed that the heterotic/type I 6-brane is T-dual to the bound state of an anti-5-brane and a KK-monopole at the self-dual radius and thus it is not supersymmetric.

The main goal of this work was to discuss the field-theory limit of  $p$ -branes in a general set-up. The starting point for the analysis was the fact that the background should be regular and finite in  $\alpha'$  units in the low-energy limit. We argued that in order for the DW/QFT dualities to be well behaved we need to restrict ourselves to  $D = 10$  D-brane intersections reduced over their relative directions (with identified harmonics). Starting with that working assumption we added new cases to the list of (conjectured) dualities between field theory and supergravity (in various dimensions) on a  $DW_{d+1} \times S_{\tilde{d}-1}$  background with a non-trivial dilaton.

One new case we discussed is the domain-wall brane in  $D = 10$  and 6 dimensions, i.e. the D8-brane and the d4-brane. A special feature of domain walls is that the ‘near-horizon’ limit  $H \rightarrow \infty$  corresponds to the asymptotic limit  $r \rightarrow \infty$  instead of  $r \rightarrow 0$  as for all the other  $p$ -branes. We found that the D8-brane is similar to the D6-brane in the sense that the field-theory and supergravity description are both valid in the infrared limit  $u = 0$ . However, an important difference is that for the D6-brane we have  $e^\phi \gg 1$  at  $u = 0$  and one must use the S-dual KK-monopole description whose near-horizon geometry is the flat Euclidean 4-space  $\mathbb{E}^4$  without a boundary [46]. On the other hand, for the D8-brane we have  $e^\phi \ll 1$  at  $u = 0$  and one can stay in 10 dimensions. This leads to a duality between  $D = 10$  (massive) supergravity and (the large- $N$  limit of)  $D = 9$  Yang–Mills. This duality seems unlikely and the D8-brane is at present not well understood. Due to its relation with the M9-brane [15] this case clearly deserves further study.

In this work we did not discuss the boundary field theories in detail except for the extreme black holes, where we constructed the (generalized conformal) quantum mechanics Hamiltonian. An interesting case is the d0-brane in  $D = 6$  dimensions. We find that in the infrared limit one must use the S-dual fundamental heterotic 0-brane description. We argued that the curvature singularity of this fundamental 0-brane, in correspondence with the curvature singularity of the fundamental  $D = 10$  string, is resolved by a free (conformal) quantum mechanics model (see section 5). It would be interesting to study more precisely the mechanism behind this curvature resolution.

Following the conjecture of [35] we considered the  $n$ -particle Calogero model as the conformal quantum mechanics model describing  $n$  stacked black holes. We pointed out that the ‘horizontal’  $SL(2, \mathbb{R})$  symmetry of the Calogero model can be extended to a Virasoro algebra. It would be interesting to see whether this Virasoro symmetry can lead to any new insight into the microscopic description of black holes.

The results of this paper have been obtained using a general set-up. Our hope is that the present work will be a convenient starting point for investigating in more detail the validity of the different conjectured DW/QFT dualities. Several issues involved in these conjectures have not been discussed here. For instance, in some cases the worldvolume QFT involved in the

DW/QFT duality is non-renormalizable. This indicates, for particular regions in the parameter space, the presence of extra non-decoupled degrees of freedom similar to those discussed in [22]. In this paper we discussed duality conjectures which can be related to *delocalized* Dp-brane intersections. Due to this we have not been very precise about the vacuum structure of the dual quantum field theory. To improve this one should in some cases consider *localized* intersections as discussed in [25, 26, 47, 48]. We hope to report on this and other issues in the near future.

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### Appendix A. Charge conventions

In this appendix we specify our charge conventions. In  $D = 10$  we use the convention  $l_s^2 = \alpha'$  which is understood to be replaced by the Planck length  $l_p$  in other dimensions.

The charge of a  $p$ -brane is defined by

$$\begin{aligned}\mu_p &= \frac{1}{\sqrt{16\pi G_D}} \int_{S_{\tilde{d}+1}} F_{\tilde{d}+1} \\ &= \frac{\Omega_{\tilde{d}+1} Q}{\sqrt{16\pi G_D}},\end{aligned}\tag{A1}$$

where the angular volume of a sphere at transverse infinity of the brane is given by

$$\Omega_{\tilde{d}+1} = \frac{2\pi^{(\tilde{d}+2)/2}}{\Gamma(\frac{1}{2}(\tilde{d}+2))}.\tag{A2}$$

Expressing the BPS relation between charge and mass in terms of the flux  $\mu_p$  and the effective tension  $\tau_p$ , we have for  $N$  coincident branes

$$\mu_p = N\sqrt{16\pi G_D} \tau_p.\tag{A3}$$

When we define a basic charge  $q$  with a (natural) unit of  $[L]^{-1}$  as

$$Nq \equiv \frac{\Omega_{\tilde{d}+1} V_p Q}{16\pi G_D},\tag{A4}$$

where  $V_p$  is the spatial volume of the  $p$ -brane, and use that  $\tau_p = m/V_p$ , then the BPS relation in terms of the quantities  $m$  and  $q$  naturally reads  $m = q$ .

Using scaling arguments (see appendix B) we find for  $\tau_p$  in the string frame

$$\tau_p = \frac{2\pi}{(2\pi l_s)^d} g_s^{-k},\tag{A5}$$

with

$$k = \frac{a}{2} + \frac{2d}{D-2}.\tag{A6}$$

We have for Newton's constant in  $D$  dimensions:

$$16\pi G_D = \frac{(2\pi l_s)^{D-2}}{2\pi} g_s^2. \quad (\text{A7})$$

Using the above we can express  $Q$  in terms of  $g_s$  and  $l_s$

$$Q = \frac{(2\pi l_s)^{\tilde{d}}}{\Omega_{\tilde{d}+1}} N g_s^{2-k}. \quad (\text{A8})$$

We can relate  $Q$  to  $r_0$  by

$$Q = \tilde{d} r_0^{\tilde{d}}. \quad (\text{A9})$$

Finally, it is convenient to define parameters  $L$  and  $d_p$  by

$$d_p = \frac{(2\pi)^{\tilde{d}}}{\tilde{d} \Omega_{\tilde{d}+1}}, \quad (\text{A10})$$

$$L \equiv \frac{r_0}{l_s} = (d_p N g_s^{2-k})^{1/\tilde{d}}. \quad (\text{A11})$$

## Appendix B. The sigma model metric

In this paper we have been working with two kinds of metric: the Einstein metric  $g_E$  and the regular ‘dual-frame’ metric  $g_D$ . The Einstein metric is defined as the metric frame in which there is no dilaton factor in front of the Einstein term, like in (14). The regular metric was defined in (23) and is the metric frame in which the  $p$ -brane solution (16) has a regular near-horizon metric, like in (29).

In this section we consider a third metric frame: the ‘sigma model’ metric  $g_\sigma$  [49]. Given a  $\tilde{d}$ -form gauge potential  $C$ , it is defined as the metric in which the sigma model action containing the  $\tilde{d}$ -form potential  $C$  has a dilaton-independent effective brane tension  $\tau_{\tilde{d}-1}$ . To be more precise, consider the action

$$S^{\text{total}} = S_D + I_D, \quad (\text{B1})$$

where  $S_D$  is the target space action given in (14) and  $I_D$  is the following sigma model action

$$I_D = T_{\tilde{d}-1} \int d^{\tilde{d}}\xi \left\{ \sqrt{\det \partial_i X^M \partial_j X^N g_{\sigma, MN} + \varepsilon^{i_1 \dots i_{\tilde{d}}} \partial_{i_1} X^{M_1} \dots \partial_{i_{\tilde{d}}} X^{M_{\tilde{d}}} C_{M_1 \dots M_{\tilde{d}}} + \dots} \right\}, \quad (\text{B2})$$

with  $T_{\tilde{d}-1}$  the brane tension parameter. We have only indicated the standard Nambu–Goto term and the leading term of the Wess–Zumino contribution. The effective brane tension  $\tau$  in the Einstein frame is given by

$$\tau_{\tilde{d}-1} = T_{\tilde{d}-1} [\Omega(\phi)]^{\tilde{d}/2} \quad (\text{B3})$$

and is independent of the dilaton only in the sigma model metric  $g_\sigma$  given by

$$g_\sigma = \Omega(\phi) g_E. \quad (\text{B4})$$

To determine the scale factor  $\Omega(\phi)$  we follow the procedure of [49] and require that the total action (B1) scales homogeneously under the scaling transformations

$$g_E \rightarrow \alpha g_E, \quad e^\phi \rightarrow \beta e^\phi, \quad C_{\tilde{d}} \rightarrow \lambda^{\tilde{d}} C_{\tilde{d}}. \quad (\text{B5})$$



Requiring that each term in the action scales with the same factor we find

$$\Omega \rightarrow \frac{\lambda^2}{\alpha} \Omega, \quad \alpha^{D/2-1} = \lambda^{\tilde{d}} = \alpha^{D/2-1-\tilde{d}} \beta^{-a} \lambda^{2\tilde{d}}, \quad (\text{B6})$$

from which we deduce that

$$\Omega = e^{(a/\tilde{d})\phi}, \quad \alpha = \lambda^{2\tilde{d}/(D-2)}, \quad \beta = \lambda^{2d\tilde{d}/(D-2)a}. \quad (\text{B7})$$

We thus find, in particular, that

$$g_\sigma = e^{(a/\tilde{d})\phi} g_E. \quad (\text{B8})$$

Comparing with the definition (23) of the regular ‘dual-frame’ metric we conclude that

$$g_\sigma = g_D. \quad (\text{B9})$$

In terms of  $g_\sigma$  the precise scaling transformations are given by

$$\begin{aligned} g_\sigma &\rightarrow \lambda^2 g_\sigma, \\ e^\phi &\rightarrow \lambda^{2d\tilde{d}/(D-2)a} e^\phi, \\ C_{\tilde{d}} &\rightarrow \lambda^{\tilde{d}} C_{\tilde{d}}, \end{aligned} \quad (\text{B10})$$

such that

$$S^{\text{total}} \rightarrow \lambda^{\tilde{d}} S^{\text{total}}. \quad (\text{B11})$$

Note that the sigma model metric  $g_\sigma$  is defined independently of any solution but that the definition of the regular ‘dual-frame’ metric  $g_D$  is related to a particular solution. In the identity (B9) it is understood that  $g_D$  is related to a  $p$ -brane solution (in the magnetic formulation), whereas  $g_\sigma$  refers to a sigma model referring to the dual  $(\tilde{d} - 1)$ -brane.

We conclude that a brane moving in a background of its dual brane does not see any spacetime singularity. For instance, a D2-brane in 10 dimensions that probes a (dual) D4-brane will enter an anti-de Sitter space in the near-horizon region. On the other hand, probing a D4-brane with a (non-dual) D1-string, that couples to the string metric, leads to a singular geometry. Only for the self-dual D3-brane all frames are equivalent and one always sees a regular geometry. Only the D3-brane has a zero dilaton, for all other  $Dp$ -branes the dilaton is singular indicating a strongly or weakly coupled region corresponding to a free-field theory either in the UV or IR fixed points.

We finally note that there is yet a third kind of metric: the usual string frame metric  $g_S$ . It is defined as the metric in which the Einstein term is multiplied by a dilaton factor  $e^{-2\phi}$  and the dilaton kinetic term carries a factor of 4 in front. In terms of the Einstein metric  $g_E$  it is given by<sup>†</sup>

$$g_S = e^{[4/(D-2)]\phi} g_E. \quad (\text{B12})$$

The string metric  $g_S$  coincides with the sigma model metric  $g_\sigma$  and/or the regular metric  $g_D$  if

$$\frac{4}{D-2} = \frac{a}{\tilde{d}}, \quad (\text{B13})$$

which includes, for example, the NS-5-brane for which  $D = 10$ ,  $a = 1$ ,  $\tilde{d} = 2$ .

<sup>†</sup> The Einstein frame (14) used, ensures that we obtain the string frame, after the conformal transformation as defined in (B12), with the correct factor of 4 in front of the dilaton kinetic term in all dimensions.

### Appendix C. Glossary of symbols used

Due to our general set-up this paper contains many symbols. For the convenience of the reader, in this appendix we give an overview of (most of) the symbols introduced in the different sections. Standard and/or obvious symbols have been omitted. Next to a brief description we have given the equation number in which the symbol first occurred.

#### Section 1

Symbol	Short description	Equation
$d$	Dimension of $p$ -brane worldvolume	(1)
$b$	Domain-wall dilaton coupling parameter	(1)
$\epsilon$	Free parameter of harmonic solution	(3)
$\Delta_{DW}$	Domain-wall parameter invariant under reductions	(4)
$\lambda$	Radial coordinate	(10)

#### Section 2

Symbol	Short description	Equation
$a$	$p$ -brane dilaton coupling parameter	(14)
$\tilde{d}$	Dimension of dual ( $\tilde{d} - 1$ )-brane worldvolume	(15)
$\Delta$	$p$ -brane parameter invariant under reductions	(17)
$\delta$	Dual-frame dilaton parameter	(24)
$\gamma$	Dual-frame dilaton kinetic parameter	(24)
$u$	AdS radial coordinate and holographic energy scale	(29)
$\mathcal{R}$	Dimensionless radius of AdS	(29)
$\beta$	Inverse of $\mathcal{R}$	(29)

#### Section 3

Symbol	Short description	Equation
$L$	Dimensionless parameter relating $r_0$ and $\sqrt{\alpha'}$	(50)
$g_f^2$	't Hooft coupling constant	(54)
$N$	Number of stacked $p$ -branes	(54)
$c_p$	Number relating $g_s$ to the field-theory coupling constant	(54)
$k$	Factor denoting the scaling of the $p$ -brane tension with $g_s$	(54)
$x$	Number giving the length dimension of $g_f^2$	(54)
$\tau_D$	Effective string tension in the dual frame	(61)
$g_{\text{eff}}^2$	Effective dimensionless coupling constant	(62)
$U$	D $p$ -brane energy scale of fixed stretched string lengths	(63)

## Section 5

Symbol	Short description	Equation
$\mathcal{L}$	Quantum mechanics Lagrangian	(90)
$m$	Mass of particle probing background geometry	(90)
$q$	Charge of particle probing background geometry	(90)
$\mathcal{H}$	Quantum mechanics Hamiltonian	(92)
$f$	Function in quantum mechanics Hamiltonian	(93)
$g$	Potential in quantum mechanics Hamiltonian	(93)
$\mathcal{V}$	Function denoting the deformation from ‘classical’ QM	(96)
$\mathcal{K}$	Generator of special conformal transformations	(98)
$\mathcal{D}$	Generator of dilatation transformations	(98)

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